Mirror symmetry and the Landau–Ginzburg/Calabi–Yau correspondence

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1 Introduction

Mirror symmetry is a phenomenon first observed by physicists studying string theory in the mid-1980s. It is a relationship between Calabi–Yau (CY) varieties. The original ideas of mirror symmetry were physical and not stated in a mathematically precise way. Mathematicians are working to develop a mathematical understanding of the relationship based on physicists' intuition. There are various formulations of mirror symmetry.

The Landau–Ginzberg (LG) model appears naturally when considering the defining equations of Calabi–Yau hypersurfaces or complete intersections. Physicists conjectured that there should be a correspondence between the CY geometry and the LG model in the early 1990s. However, the mathematical formulations could not be made precise until the development of the Fan–Jarvis–Ruan–Witten (FJRW) theory in 2007. After that, a great deal of progress has been made.

In this article, I will first give a simple statement of mirror symmetry, then try to explain the notion of the Landau–Ginzburg/Calabi–Yau correspondence with the help of this statement. I will also introduce recent progress in this field, and I will talk about the project of my Ph.D thesis at the end.

2 A statement of mirror symmetry and a classic example

We start from a simple mathematical statement of mirror symmetry, due to Berglund and Hübsch [1]. It establishes a cohomological identity and we refer to it as cohomological mirror symmetry. A special case of this statement is an example which inspired the whole phenomenon of mirror symmetry.

We are going to construct a pair of objects $[X_W/\widetilde{G}]$ and $[X_{W^\vee}/\widetilde{G^\vee}]$ with isomorphic cohomology groups after a 90-degree rotation of the Hodge diamond.

Consider a polynomial

$$W(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} \prod_{j=1}^{N} x_j^{m_{ij}},$$

where the matrix

$$M = \{m_{ij}\},$$

is invertible. Let

$$M^{-1} = \{m^{ij}\}$$

and take integers w_i and d such that

$$\frac{w_j}{d} = \sum_{i=1}^{N} m^{ji},$$

where $\gcd(w_1, \ldots, w_N, d) = 1$. Then W is a weighted homogeneous polynomial of degree d and weights $w_1, \ldots, w_N$. We define the set

$$X_W := \{W = 0\} \subseteq \mathbb{P}(w_1, \ldots, w_N).$$

Here $\mathbb{P}(w_1, \ldots, w_N)$ is the weighted projective space defined by

$$\mathbb{C}^N/\mathbb{C}^*, \quad \text{where } \lambda \cdot (x_1, \ldots, x_N) = (\lambda^{w_1} x_1, \ldots, \lambda^{w_N} x_N).$$

We need the non-degenerate condition

$$W = \frac{\partial W}{\partial x_j} = \cdots = \frac{\partial W}{\partial x_N} = 0 \Longleftrightarrow x_1 = \cdots = x_N = 0,$$

and the condition

$$\sum_{j=1}^{N} w_j = d,$$

or equivalently,

$$\sum_{i,j=1}^{N} m^{ij} = 1$$

to make it a Calabi–Yau orbifold, i.e. $\omega_{X_W} \simeq \mathcal{O}_{X_W}$.

We transpose M. Let

$$W^\vee(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} \prod_{j=1}^{N} x_j^{m_{ji}}.$$

Then $W^\vee$ is still non-degenerate (see [11, p. 5]) and

$$X_{W^\vee} := \{W^\vee = 0\} \subseteq \mathbb{P}(w_1^\vee, \ldots, w_N^\vee)$$

is still of CY type. Define the automorphism group

$$\mathrm{Aut}(W) := \{\mathrm{diag}(\alpha_1, \ldots, \alpha_N) \colon W(\alpha_1 x_1, \ldots, \alpha_N x_N) = W(x_1, \ldots, x_N)\},$$

and its subgroup

$$\mathrm{SL}_W := \mathrm{Aut}(W) \cap \mathrm{SL}(n, \mathbb{C}).$$

Because of the CY condition we have

$$\mathrm{diag}\left(e^{\frac{w_1}{d}2\pi i}, \ldots, e^{\frac{w_N}{d}2\pi i}\right) \in \mathrm{SL}_W.$$

Define

$$J_W := \left\langle \mathrm{diag}\left(e^{\frac{w_1}{d}2\pi i}, \ldots, e^{\frac{w_N}{d}2\pi i}\right)\right\rangle \subseteq \mathrm{SL}_W.$$

If G is a subgroup of $\mathrm{Aut}(W)$, the injection

$$i \colon G \hookrightarrow \mathrm{Aut}(W)$$

induces a surjection

$$i^* \colon \mathrm{Aut}(W)^* \twoheadrightarrow G^*,$$

where, for any group H, the group H^* denotes $\mathrm{Hom}(H, \mathbb{C}^*)$.

There is a pairing $\mathrm{Aut}(W) \times \mathrm{Aut}(W^\vee) \to \mathbb{C}^*$:

$$\left(\mathrm{diag}\left(e^{a_1 2\pi i}, \ldots, e^{a_N 2\pi i}\right), \mathrm{diag}\left(e^{b_1 2\pi i}, \ldots, e^{b_N 2\pi i}\right)\right) \mapsto \exp\left(2\pi i \sum_{i,j=1}^{N} m_{ij} a_j b_i\right). \tag{1}$$

We can check it is non-degenerate and get

$$\mathrm{Aut}(W)^* = \mathrm{Aut}(W^\vee).$$

We can then define

$$G^\vee := \ker(i^*) \subseteq \mathrm{Aut}(W^\vee).$$

From pair (1) we see

$$J_W^\vee = \mathrm{SL}_{W^\vee}$$

and

$$\mathrm{SL}_W^\vee = J_{W^\vee}.$$

By the contravariant relation, if

$$J_W \subseteq G \subseteq \mathrm{SL}_W,$$

then

$$J_{W^\vee} \subseteq G^\vee \subseteq \mathrm{SL}_{W^\vee}.$$

Now we have constructed two quotient varieties X_W/G and $X_{W^\vee}/G^\vee$. They are singular in general, but have orbifold structures, i.e. locally look like $\mathbb{C}^n$ quotient by a finite group. So we consider the Chen–Ruan orbifold cohomology groups.

Definition 2.1. Let $[X/G]$ be a complex orbifold. We define the *Chen–Ruan orbifold cohomology groups*

$$\mathrm{H}^d_{\mathrm{orb}}([X/G],\mathbb{C}) := \bigoplus_{g\in G} \mathrm{H}^{d-2a_g}(X_g/G,\mathbb{C})$$

and the *Chen–Ruan orbifold Dolbeault cohomology groups*

$$\mathrm{H}^{p,q}_{\mathrm{orb}}([X/G],\mathbb{C}) := \bigoplus_{g\in G} \mathrm{H}^{p-a_g,q-a_g}(X_g/G,\mathbb{C}),$$

where X_g is the set of points fixed by g in X, and the number a_g is defined as follows: take $x \in X_g$, then the tangent map induced by g makes g an element of $\mathrm{GL}(\mathrm{T}_x X)$. Write it as a diagonal matrix $\mathrm{diag}\left(e^{\frac{\alpha_1}{d}2\pi i}, \dots, e^{\frac{\alpha_N}{d}2\pi i}\right)$ where $\frac{\alpha_i}{d} \in [0,1)$. We define

$$a_g := \sum_{i=1}^{N} \frac{\alpha_i}{d}.$$

Remark 2.2. Definition 2.1 is a simplified version. In fact, it only makes sense when G is alelian, the set of elements $g \in G$ such that X_g is non-empty is finite, and each X_g is connected. The cases which we will consider satisfy all of these conditions.

Remark 2.3. The Chen–Ruan orbifold cohomology of X is isomorphic to the ordinary cohomology of the crepant resolution $\tilde{X}$ of X when $\tilde{X}$ exists. A resolution is crepant if it does not affect the canonical class. So if X is Calabi–Yau, then so is $\tilde{X}$.

Now we can state our theorem.

Theorem 2.4. *Under all above conditions, we have*

$$H^{p,q}_{orb}([X_W/\widetilde{G}],\mathbb{C}) \cong H^{N-2-p,q}_{orb}([X_{W^\vee}/\widetilde{G^\vee}],\mathbb{C}),$$

where

$$\widetilde{G} = G/J_W, \quad \widetilde{G^\vee} = G^\vee/J_{W^\vee}.$$

Remark 2.5. We quotient by the action of $\widetilde{G}$ and $\widetilde{G^\vee}$ instead of G and $G^\vee$, in order to make the action faithful.

Example 2.6. We consider the special case where

$$W = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5$$

and

$$G = J_W = \langle e^{\frac{2\pi i}{5}} \rangle = \boldsymbol{\mu}_5.$$

Then X_W is a quintic three-fold in $\mathbb{P}^4$ and $\widetilde{G}$ acts trivially on it. On the other side, $W^\vee = W$ and $X_{W^\vee}/\widetilde{G^\vee}$ is singular but has a crepant resolution Y. Their Hodge diamonds are as follows

$$h^{p,q}(Y) = \begin{matrix} &&& 1 &&& \\ && 0 && 0 && \\ & 0 && 101 && 0 & \\ 1 && 1 && 1 && 1 \\ & 0 && 101 && 0 & \\ && 0 && 0 && \\ &&& 1 &&& \end{matrix} \qquad h^{p,q}(X_W) = \begin{matrix} &&& 1 &&& \\ && 0 && 0 && \\ & 0 && 1 && 0 & \\ 1 && 101 && 101 && 1. \\ & 0 && 1 && 0 & \\ && 0 && 0 && \\ &&& 1 &&& \end{matrix}$$

3 The Landau–Ginzburg/Calabi–Yau correspondence

Theorem 2.4 was proved in [4] via the following isomorphisms

$$\begin{array}{ccc} \mathrm{H}^{p,q}_{\mathrm{orb}}([X_W/\widetilde{G}],\mathbb{C}) & & \mathrm{H}^{N-2-p,q}_{\mathrm{orb}}([X_{W^\vee}/\widetilde{G^\vee}],\mathbb{C}) \\ \Vert & & \Vert \\ [\mathbb{H}_{W,G}]_{p,q} & = & [\mathbb{H}_{W^\vee,G^\vee}]_{N-2-p,q} \end{array} \tag{2}$$

The vertical relation in (2) is the so-called Landau–Ginzburg/Calabi –Yau (LG/CY) correspondence. The upper row is called Calabi–Yau side, because it provides information about a Calabi–Yau hypersurface in the weighted projective space. The object in the lower row is defined below. It provides information about the Landau–Ginzburg model $(\mathbb{C}^N, W, G)$, where W is a G-invariant function on $\mathbb{C}^N$.

Definition 3.1. We define

$$\mathbb{H}_{W,G} := \bigoplus_{g\in G} (\operatorname{Jac} W_g)^G.$$

Here W_g is W restricted to $\mathbb{C}^N_g$, and $\mathbb{C}^N_g$ is the subspace of $\mathbb{C}^N$ which is fixed by g; $\operatorname{Jac} f$ is the Jacobian ring of a polynomial $f = f(x_1, \dots, x_k)$, defined as

$$\operatorname{Jac} f := \mathbb{C}[x_1, \dots, x_k] \Big/ \left\langle \frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_k} \right\rangle.$$

Denote the dimension of $\mathbb{C}_g^N$ by N_g. $g = \mathrm{diag}(d_1, \ldots, d_N)$ acts on $\prod_{j=1}^{N_g} x_j^{m_j-1} \in \mathrm{Jac}\, W_g$ by multiplication by $\prod_{j=1}^{N} d_j^{m_j}$.

For $\prod_{j=1}^{N_g} x_j^{m_j-1} \in \mathrm{Jac}\, W_g \subset \mathbb{H}_{W,G}$, we define its bidegree as

$$(p, q) = \left(N_g - \sum_{j=1}^{N_g} \frac{m_j w_j}{d}, \sum_{j=1}^{N_g} \frac{m_j w_j}{d} \right) + (a_g, a_g) - \left(\sum_{j=1}^{N} \frac{w_j}{d}, \sum_{j=1}^{N} \frac{w_j}{d} \right). \quad (3)$$

Remark 3.2. The horizontal relation in (2) is the Krawitz LG-to-LG mirror symmetry theorem (see [11]). We can see from the definition that things on the LG side is are easier to calculate than on the CY side.

The story doesn't end here. In fact, there are correspondences between LG and CY sides which contain much more information. Now we restrict ourself to the case $G = J_W = \boldsymbol{\mu}_d$.

The Gromov–Witten (GW) theory is about counting the number of surfaces satisfying some conditions in a given variety. The space $\mathrm{H}_{\mathrm{orb}}(X_W, \mathbb{C})$ serves as the state space of the GW theory of X_W. For a fixed genus g, the Gromov–Witten invariants

$$\langle \tau_{a_1}(\varphi_{h_1}), \ldots, \tau_{a_{n-1}}(\varphi_{h_{n-1}}), \tau_{a_n}(\varphi_{h_n}) \rangle_{g,n,\delta}^{\mathrm{GW}}$$

are numbers which depend on the classes $\varphi_{h_i} \in \mathrm{H}_{\mathrm{orb}}(X_W, \mathbb{C})$, the integers $a_i \geq 0$, and the class $\delta \in \mathrm{H}_2(X_W, \mathbb{Z})$.

On the other side, Fan–Jarvis–Ruan [8] [10] [9] developed a theory for the LG model, which is called the Fan–Jarvis–Ruan–Witten (FJRW) theory. The state space of the FJRW theory for $(\mathbb{C}^N, W, \boldsymbol{\mu}_d)$ is $\mathbb{H}_{W,\boldsymbol{\mu}_d}$. There are also FJRW invariants

$$\langle \tau_{a_1}(\phi_{m_1}), \ldots, \tau_{a_{n-1}}(\phi_{m_{n-1}}), \tau_{a_n}(\phi_{m_n}) \rangle_{g,n}^{\mathrm{FJRW}}$$

where $\phi_{m_i} \in \mathbb{H}_{W,\boldsymbol{\mu}_d}$.

In genus $g = 0$, and when W is a quintic, there are I functions I_{GW} and I_{FJRW} which contain all the information about the invariants in both theories. The function $I(z, x)$ is a $\mathrm{H}((z^{-1}))$-valued function, where H is the state space in both theories. Chiodo–Ruan showed in [3] that there exists a $\mathbb{C}[z, z^{-1}]$ linear transformation $\mathbb{U}$ from $\mathrm{H}_{\mathrm{FJRW}}$ to H_{GW} which maps I_{FJRW} to the analytic continuation of I_{GW}. This provides a correspondence between the GW theory of X_W and the FJRW theory of $(\mathbb{C}^N, W, \boldsymbol{\mu}_5)$.

Remark 3.3. In [3], they were working on restricted theories and H_{GW} and $\mathrm{H}_{\mathrm{FJRW}}$ are subspaces of $\mathrm{H}_{\mathrm{orb}}(X_W, \mathbb{C})$ and $\mathbb{H}_{W,\boldsymbol{\mu}_d}$.

4 Relationship with the Orlov equivalence

Chiodo–Iritani–Ruan extended above correspondence to a general weighted homogeneous polynomial W in [2]. Moreover, they described a relationship between this correspondence and the Orlov equivalence. To understand Orlov equivalence, we should know about matrix factorizations.

Definition 4.1. A graded matrix factorization of a polynomial W is a collection $(E^i, \delta_i)_{i\in\mathbb{Z}}$ of finitely generated graded free modules E^i over the polynomial ring and degree-zero homomorphisms $\delta_i \in \mathrm{Hom}_{\text{gr-}R}(E^i, E^{i+1})$ such that the sequence

$$\dots \xrightarrow{\delta_{-1}} E^0 \xrightarrow{\delta_0} E^1 \xrightarrow{\delta_1} E^2 \xrightarrow{\delta_2} E^3 \xrightarrow{\delta_3} \dots$$

is 2-periodic up to the shift of grading

$$E^{i+1} = E(d), \quad \delta(i+1) = \delta_i(d),$$

and such that for all i, we have

$$\delta(i+1) \circ \delta(i) = W \cdot \mathrm{id}_{E^i} \colon E^i \to E^i(d).$$

Remark 4.2. The reason why it is called matrix factorization of W is

$$\begin{pmatrix} 0 & \delta_1 \\ \delta_0 & 0 \end{pmatrix}^2 = W \cdot \mathrm{id}_{E^0 \oplus E^1}.$$

The category $\mathrm{MF}^{\mathrm{gr}}_{\boldsymbol{\mu}_d}(W)$ is a category whose objects are graded matrix factorizations of W, and morphisms between two matrix factorizations can be represented by usual morphisms between complexes. The lower index $\boldsymbol{\mu}_d$ stands for $\boldsymbol{\mu}_d$-equivariant. It is automatically satisfied.

Under the Calabi–Yau condition, Orlov [13] constructed equivalences of categories

$$\Phi_l : \mathrm{MF}^{\mathrm{gr}}_{\boldsymbol{\mu}_d}(W) \to \mathrm{D}^{\mathrm{b}}(X_W),$$

indexed by an integer l, where $\mathrm{D}^{\mathrm{b}}(X_W)$ is the derived category of coherent sheaves on X_W.

The Chern characters of graded matrix factorization take value in $\mathrm{H}_{\mathrm{FJRW}}$, and the Chern characters take value in H_{GW}. The relationship between the LG/CY correspondence and the Orlov equivalence can be roughly described by the commutative diagram

$$\begin{array}{ccc} \mathrm{MF}^{\mathrm{gr}}_{\boldsymbol{\mu}_d}(W) & \xrightarrow{\Phi_l} & \mathrm{D}^{\mathrm{b}}(X_W) \\ \downarrow{\scriptstyle \mathrm{ch}} & & \downarrow{\scriptstyle \mathrm{ch}} \\ \mathrm{H}_{\mathrm{FJRW}} & \xrightarrow[\mathbb{U}_l]{} & \mathrm{H}_{\mathrm{GW}} \end{array} .$$

Remark 4.3. The lower index l in $\mathbb{U}_l$ comes from taking analytic continuation alone different paths.

5 The Geometric Invariant Theory point of view

The LG/CY correspondence can be seen as a result of variations of stability conditions in Geometric Invariant Theory (GIT). We recall the following.

Definition 5.1. Let L be a line bundle on a complex variety X with projection $\pi\colon L \to X$, and let G be a reductive algebraic group with an action σ on X. A G-*linearisation* of L is an extension of the action σ on X to an action $\overline{\sigma}$ on L such that the diagram

$$\begin{array}{ccc} G \times L & \xrightarrow{\overline{\sigma}} & L \\ {\scriptstyle \mathrm{id}\times\pi}\downarrow & & \downarrow{\scriptstyle \pi} \\ G \times X & \xrightarrow[\sigma]{} & X \end{array}$$

commutes, and G acts linearly on each fiber. We write L_χ for the line bundle with a G-linearisation χ.

Definition 5.2. Let L_χ be a G-linearized line bundle on X. We define

1. *The set of semistable points* $X^{ss}_G(L_\chi)$: a point $x \in X$ is said to be semistable if there exists a section $s \in \mathrm{H}^0(X, L_\chi^{\otimes n})$ for some $n > 0$, such that $s(x) \neq 0$, and s is G-invariant.

2. *The set of stable points* $X^{s}_G(L_\chi)$: a point $x \in X$ is said to be stable if it is semistable and the stabilizer G_x is finite.

3. *The set of unstable points* $X^{us}_G(L_\chi)$: a point $x \in X$ is said to be unstable if it is not semistable.

Definition 5.3. The GIT quotient $[X /\!\!/_\chi G]$ is defined to be the orbifold

$$[X /\!\!/_\chi G] = [X^{\mathrm{ss}}_G(L_\chi)/G].$$

We focus on the case where the base space is a finite-dimensional vector space $V \cong \mathbb{C}^n$ and $G = (\mathbb{C}^*)^k$. Since V is contractible, the line bundle L on V is always the trivial bundle $\mathcal{O}_V$, whose total space is $V \times \mathbb{C}$. Then the G-linearisation can be determined by a character $\theta\colon G \to \mathbb{C}^*$, such that

$$g \cdot (x, l) = (g.x, \theta(g)l).$$

Example 5.4. Consider the $G = \mathbb{C}^*$ action on the vector space $V = \mathbb{C}^N_{\underline{x}} \times \mathbb{C}_p$:

$$(x_1, \ldots, x_N, p) \mapsto (\lambda^{w_1} x_1, \ldots, \lambda^{w_N} x_N, \lambda^{-d} p), \lambda \in G$$

satisfying the CY condition

$$\sum_{i=1}^{N} w_i = d.$$

Positive phase. We take the character $\theta_1 : G \to \mathbb{C}^*$ defined by

$$\theta_1(\lambda) = \lambda^k, \quad k > 0.$$

Then

$$V_G^{\mathrm{ss}}(L_{\theta_1}) = (\mathbb{C}^N \backslash \{\mathbf{0}\}) \times \mathbb{C}$$

and

$$[V/\!\!/_{\theta_1}\mathbb{C}^*] = [(\mathbb{C}^N \backslash \{\mathbf{0}\}) \times \mathbb{C}/\mathbb{C}^*],$$

which is the line bundle $\mathcal{O}(-d)$ over $\mathbb{P}(\underline{w})$.

Negative phase. Now we take the character $\theta_2 : G \to \mathbb{C}^*$ defined by

$$\theta_2(\lambda) = \lambda^{-k}, \quad k > 0.$$

In this case

$$V_G^{\mathrm{ss}}(L_{\theta_2}) = \mathbb{C}^N \times \mathbb{C}^*$$

and

$$[V/\!\!/_{\theta_2}\mathbb{C}^*] = [\mathbb{C}^N \times \mathbb{C}^*/\mathbb{C}^*] = [\mathbb{C}^N/\boldsymbol{\mu}_d].$$

An easy computation shows that

$$\mathrm{H}^*_{\mathrm{orb}}([V/\!\!/_{\theta_1}\mathbb{C}^*], \mathbb{C}) = \mathrm{H}^*_{\mathrm{orb}}([V/\!\!/_{\theta_2}\mathbb{C}^*], \mathbb{C}).$$

Remark 5.5. In my mémoire de M2, I computed several examples, including one with $k = 2, N = 7$, to show that under the CY condition, different linearisations give different GIT quotient, but they have isomorphic Chen–Ruan cohomology groups.

Example 5.4 is a system without potential function. We will add a potential function on it. Let $W(\underline{x})$ be a weighted homogeneous polynomial with weights $\underline{w}$ and degree d, then $\widetilde{W}(\underline{x}, p) := pW(\underline{x})$ is a $\mathbb{C}^*$-invariant function on $\mathbb{C}^N_{\underline{x}} \times \mathbb{C}_p$. In the positive phase, we have the line bundle $\mathcal{O}(-d)$ over $\mathbb{P}(\underline{w})$ with the induced function $\widetilde{W}$, this corresponds to the CY side in section 3. In the negative phase, we have the quotient $[\mathbb{C}^N/\mu_d]$ with the induced function $\widetilde{W}$, this corresponds to the LG model in section 3.

Corti–Coates–Iritani–Tseng [6] gave a correspondence between the two sides without the potential functions, which is a weak form of crepant transformation conjecture (CTC). In [12], Lee–Priddis–Shoemaker deduced the LG/CY correspondence for a special type of polynomial from this correspondence.

6 The Calabi–Yau complete intersection case

Let Z be a Calabi–Yau complete intersection (CYCI) in a weighted projected space $\mathbb{P}(\underline{w})$, defined by M weighted homogeneous polynomials $W_i \colon \mathbb{C}^N \to \mathbb{C}$ of degree d_i. Consider the $G = \mathbb{C}^*$ action on the vector space $V = \mathbb{C}^N_{\underline{x}} \times \mathbb{C}^M_{\underline{p}}$:

$$\lambda \cdot (x_1, \ldots, x_N, p_1, \ldots, p_M) = (\lambda^{w_1} x_1, \ldots, \lambda^{w_N} x_N, \lambda^{-d_1} p_1, \ldots, \lambda^{-d_M} p_M).$$

Together with the potential $W(\underline{x}, \underline{p}) := \sum p_i F_i(\underline{x})$, we get a GIT system with potential.

In this case, there are also two possible GIT quotients. One of them yields the Gromov–Witten theory of Z, the other one yields a Landau–Ginzburg-type theory.

Clader–Ross proved in [5] that the genus zero Gromov–Witten theory of Z is equivalent to the genus zero Landau–Ginzburg-type theory (constructed in [7]), under some "Fermat-type" conditions. Their proof is motivated by ideas introduced in [12], i.e. deduced from a correspondence without potential functions.

The first goal of my Ph.D thesis is to clarify the CICY case as much as the hypersurface case. For example, I will try to construct an analog of Orlov equivalence for CICY and show its connection with the correspondence given by Clader–Ross. My long term goal is to clarify gauged linear sigma model (GLSM).

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