Impact of basis volatility on vanille product pricing

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Introduction au domaine de recherche

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Chapter 1

Introduction to the interest rate world

1.1 Overview about Interest rate world

The concept of interest rate belongs to our every-day life and has entered our minds as something familiar we know how to deal with. When depositing a certain amount of money in a bank account, everybody expects that the amount grows as time goes by. The fact that lending money must be rewarded somehow is indeed common knowledge and wisdom. For big company, interest rate instruments play an important role to refinance their projects. Interest rates targets are also a vital tool of monetary policy and are taken into account when dealing with variables like investment, inflation, and unemployment.

In the interest rate world, for any given currency there are many types of interest rates which are traded in the market. We will consider some main reference rates :

(i) The treasury rate (Treasury bonds) are the instruments used by a government to borrow in its own currency. It is usually assumed that there is no chance that a government will default on an obligation denominated in its own currency (by printing more money). Treasury rates are therefore totally risk-free rates in the sense that an investor who buys a treasury bill or treasury bond is certain that interest and principal payments will be made as promised.

(ii) LIBOR, London InterBank Offer Rate, is the average interest rate estimated by leading banks in London that they would be charged if borrowing from other banks. A bank must satisfy certain creditworthiness criteria in order to be able to accept a Libor quote from another bank and receive deposits from that bank at Libor. Typically it must have to have a AA credit rating. *AA-rated financial institutions regard Libor as their short-term opportunity cost of capital.*

1.2 Basic notions

1.2.1 Discount factor

Discounting is a financial mechanism in which a debtor obtains the right to delay payments to a creditor, for a defined period of time, in exchange for a charge or fee. The discount, or charge, is simply the difference between the original amount owed in the present and the amount that has to be paid in the future to settle the debt.

Mathematically, if we note $\mathrm{DF}(t,T)$ is the discount factor between t and T, then at t one invests $\mathrm{DF}(t,T)$(dollar) then one will receive 1 dollar at T. In other way, the discount factor represents the cost of future cash flow at present and banks consider the level of the discounting curve as the cost of funding today. And that is why in pre-crisis period, the USD Libor (3-month) was the market convention for discounting all trades independently of the counterparty. Let us see the relation between the Libor rate and discount factor in this case.

One notes Libor(t,T) the Libor rate, known at t, between t and T, then if one invests 1 dollar at t then he will receive $1+(T-t)\text{Libor}(t,T)$ at T. In order to avoid the arbitrage, we should have

$$\frac{1}{\text{DF}(t,T)} = 1+(T-t)\text{Libor}(t,T),$$

which gives us the relationship between discount factor and Libor and give us the method that we can obtain the discounting curve from the Libor curve. Libor rates are not totally free of credit risk. There is a small chance that a AA-rated financial institution will default on a Libor loan. After the bankruptcy of Lehman Brothers, the banking system become more risky with a lot of changes. Specially in order to count in the counter-party risk, many banks use the collateral system to secure its transactions with other financial institutions. Intuitively, the cost of funding/capital for a high risky counter-party is not the same with the cost for a less risky counter-party. This fact means that the discount factor curve is no more the same as discount curve derived from the Libor curve. The method to obtain the discount curve now becomes more and more complex that we omit in this text. The basis is the spread between these two curves, which raises a new source of risk in pricing interest rate instruments, called basis risk. And in my internship at BNP, my project is to study this impact of risk on some type of vanilla products.

1.2.2 Numeraire and Forward

Numeraire

We first recall the definition of *price* as "a ratio of one asset A dominated in another asset B, when doing a fair trade of A for B" . For example, if 1 apple costs the same as 3 oranges, then apple denominated by orange has a price of 3. The fair trade means that the price is fixed by offers and demands.

A numeraire asset is any asset that always has a positive intrinsic value so that it can be used as the domination to price other assets, for example: Dollar, Euro or gold,...

Forward

In the interest rates world, a floating cash flow is an instrument that fixes its nominal value on some index I at a future time T_f and pay $I(T_f)$ units of asset N. Then the floating cash flow is well defined only if one specifies both the nominal value and the unit asset. For example, we consider a FRA, which is a contract involving three time instants:

- The current time t
- The expiry time T
- The maturity time S

The contract gives its holder an interest rate payment for the period between T and S. At the maturity S, a fixed payment based on a fixed rate K is exchanged against a floating payment based on the Libor rate resetting in T and with maturity S.

Then the numeraire here is the discount factor at S and the floating cash flow is Libor(T,S), which is fixed at T.

In finance, *Forward* can be considered as the exchange of 2 assets in the future at a strike K that give the present value of the contract is zero at today. Other way, the forward is the fair price at present of the cash flow dominated in the numeraire

$$Forward = \frac{\text{PV}(Floating\_cash)}{\text{PV}(Numeraire)}.$$

Noting PV means present value. In the case of of FRA, we will have

$$Fra(t,T,S) = \frac{PV(Libor(T,S)}{PV(\,\mathrm{DF}(S))} = \frac{PV(1-\,\mathrm{DF}(T,S))}{PV((S-T)\,\mathrm{DF}(T,S))} = \frac{\mathrm{DF}(t,T)-\,\mathrm{DF}(t,S)}{(S-T)\,\mathrm{DF}(t,S)}.$$

1.2.3 Swap rate and CMS rate

Now we have all the concepts to introduce the forward Swap rate, the main underlying of interest rate market. A prototypical Payer Interest rate Swap is a contract that exchanges payments between two differently indexed legs, starting from a future time instant. At every instant T_i in a prespecified of dates $T_1, ..., T_n$:

- The fixed leg pays out the amount $N\tau_i K$ corresponding to a fixed interest rate K, a nominal value N and a year fractionτ_i between T_{i-1} and T_i.
- The floating leg pays the amount $N.\tau_i.Libor(T_{i-1}, T_i)$, corresponding to the interest rate $Libor(T_{i-1}, T_i)$, reseting at the previous instant T_{i-1} for the maturity given by the current payment instant T_i.

The numeraire corresponding with the swap cash is called Level, and given by :

$$\mathrm{LVL}(t, T_0, T_n) = \sum_1^n \delta_i\,\mathrm{DF}(t, T_i)$$

where $\delta_i = T_i - T_{i-1}$. This numeraire presents the payment method of a swap. And the floating cash flow of Swap as sum of s string of floating cash flow of FRA, then we can define the forward swap rate is defined as :

$$\mathrm{Swap}(t, T_0, T_n) = \frac{\sum_0^{n-1} \delta_i\,\mathrm{FRA}(t, T_i, T_{i+1})\,\mathrm{DF}(t, T_i)}{\mathrm{LVL}(t, T_0, T_n)}$$

From the expression of FRA before we can easily obtain:

$$\mathrm{Swap}(t, T_0, T_n) = \frac{\mathrm{DF}(t, T_0)-\,\mathrm{DF}(t.T_n)}{\mathrm{LVL}(t, T_0, T_n)}$$

1.2.4 Pricing model and numeraire change

A *Pricing model* is at the most fundamental level- a set prices $\{P_i(t,\omega\}$ for a set of cash flows $\{CF_i(t,\omega)\}$ defined on a (greatly simplified) set of future events $\{\omega\}$, such as curve moves or stock price changes. One of the most important that a Pricing model should verify is the condition of non-arbitrage : any linear combination of the cash flows that creates a non-negative cash flow regardless event ω will have non-negative price. Mathematically that means if the price process of any asset $P(t)$ denominated in the numeraire asset $N(t)$ then it should exist a probability measure Q^N which under this probability the process $P(t)/N(t)$ is a martingale

$$\frac{P(t)}{N(t)} = E^{Q_N}(\frac{P(T)}{N(T)}|F_t)$$

For example, if we consider our numeraire asset is the capital available for the bank at date T, ie $N(t) = \,\mathrm{DF}(t,T)$, then the model that we should use to evaluate the price in this numeraire is the probability "forward neutre", Q^T. The existence of this probability is omit here. Then for every cash flow $CF(t)$ we have

$$\mathrm{CF}(t) = \,\mathrm{DF}(t,T).\mathrm{E}^T[CF(T)|F_t].$$

For the numeraire "Level", we obtain the probability "Level",$Q^{\,\mathrm{LVL}}$

$$\mathrm{CF}(t) = \,\mathrm{LVL}(t, T_0, T_n).\mathrm{E}^{\,\mathrm{LVL}}[\frac{CF(T)}{\mathrm{LVL}(T_0, T_0, T_n)}|F_t]$$

Then we obtain the numeraire change formula

$$\mathrm{E}^{T}[CF(T)|F_t] = \frac{\mathrm{LVL}(t,T_0,T_n)}{\mathrm{DF}(t,T)}.\mathrm{E}^{\,\mathrm{LVL}}[\frac{CF(T)}{\mathrm{LVL}(T_0,T_0,T_n)}|F_t]. \tag{1.1}$$

1.3 Convexity adjustment

In finance it is really common to have different floating cash flows defined on the same underlying index but with different unit assets. The natural question is if there is a difference between the forwards corresponding to these numeraires and how one can measure it. The difference is called convexity adjustment. In other word, the convexity adjustment happens when one uses a wrong pricing model, ie under wrong probability measure.

In this text, we consider the Swap rate as our index to fix the floating cash flow and the two numeraire

- The Discounting numeraire, $\mathrm{DF}(t,T_0)$ which gives us the CMS rate that is paid only once at the first payment date

$$\mathrm{CMS}(0,T_0,T_n) = \mathrm{E}^{T_0}[\mathrm{Swap}(T_0,T_0,T_n))] \tag{1.2}$$

- The Level numeraire, $\mathrm{LVL}(t,T_0,T_n)$ which gives us the forward swap rate, that can be calculated as above or

$$\mathrm{E}^{\,\mathrm{LVL}}[\mathrm{Swap}(T_0,T_0,T_n))] = \mathrm{E}^{\,\mathrm{LVL}}[\frac{1-\ \mathrm{DF}(T_0,T_n)}{\mathrm{LVL}(T_0,T_0,T_n)}] = \frac{\mathrm{DF}(0,T_0)-\ \mathrm{DF}(0,T_n)}{\mathrm{LVL}(t,T_0,T_n)} = \mathrm{Swap}(0,T_0,T_n)$$

In the next section, we will try to construct a model in that we can price the CMS rate from the distribution of Swap under the probability Level.

1.3.1 CMS convexity adjustment

In this section, we suppose that the discount factor curve and the Libor curve are the same. We will construct a vanilla model with only input is the distribution of the swap under the probability Level, which is extracted from the Swaption market. In order to simplify the notion, we note :

$$S = \mathrm{Swap}(T_0,T_0,T_n);\ S_0 = \mathrm{Swap}(0,T_0,T_n);\ \mathrm{LVL}_0 = \frac{\mathrm{LVL}(0,T_0,T_n)}{\mathrm{DF}(0,T_0)}$$

The convexity adjustment is calculated as :

$$\mathrm{CMS}(0,T_0,T_n) = \mathrm{E}^{T_0}(S) = \mathrm{E}^{\,\mathrm{LVL}}\big(S\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}\big)$$

Then the CMS convexity adjustment are driven by the covariance between the underlying swap rate and its associated annuity. Consequently, it depends on the following components: the volatilities of the forward swap rate and the level payment and their correlation. But we can project the Level on the Swap rate as follow:

$$\mathrm{CMS}(0,T_0,T_n) = \mathrm{E}^{T_0}(S) = \mathrm{E}^{\,\mathrm{LVL}}\big[SM(S)\big]$$

where the annuity mapping function $M(x)$ is defined as:

$$M(x) = \mathrm{E}^{\,\mathrm{LVL}}[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}|S=x]$$

The crux of vanilla modeling for CMSs lies in specifying the annuity mapping functions. The question is what are conditions that the mapping M should satisfy. Firstly, because in the vanilla model we just specify the distribution of the swap rate and we still want to capture all the risks and in particular all the volatility and correlation sensitivities, that a full term structure model would show. Then the mapping M should contain these information. Aside from capturing all these volatility and correlation dependencies, there are a number of no-arbitrage requirements that annuity mapping functions must satisfy as

$$\mathrm{E}^{\,\mathrm{LVL}}[M(S)] = 1.$$

And by changing the measure of the conditional expectation we obtain

$$\begin{aligned} M(S) &= \mathrm{E}^{\,\mathrm{LVL}}[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0, T_0, T_n)}|S] \\ &= \mathrm{LVL}_0 \frac{1}{E^T[\,\mathrm{LVL}|S]} \\ &= \mathrm{LVL}_0 \frac{1}{\sum_1^n E^T[\delta\, \mathrm{DF}(T_0, T_i)|S]}. \end{aligned}$$

Recall that in this section, we suppose that the Libor curve and funding curve are the same (there is no spread between these two), then discount factors can be easily project on the forward swap rate and we obtain

$$M(S) = \frac{1}{\alpha\, \mathrm{CLVL}(S)},$$

where

$$\mathrm{CLVL}(t, T_0, T_n, R) = \sum_{i=1}^{n} \frac{\delta_i}{(1+R)^{T_i - t}}.$$

And the coefficient α is defined from the non-arbitrage condition

$$\alpha = \mathrm{E}^{\,\mathrm{LVL}}[\frac{1}{\mathrm{CLVL}(S)}].$$

Hence, the CMS is given by

$$\mathrm{CMS}(0, T_0, T_n) = \mathrm{E}^{T_0}(S) = \mathrm{E}^{\,\mathrm{LVL}}[S \frac{1}{\alpha\, \mathrm{CLVL}(S)}] = \mathrm{E}^{\,\mathrm{LVL}}[\Phi(S)].$$

In finance this kind of payoff can be calculated by the Carr formula

$$E[\Phi(S)] = \Phi(H) + \Phi'(H)E[S - H] + \int_0^H \Phi''(K)P(K)dK + \int_H^\infty \Phi''(K)C(K)dK,$$

where the $C(K)$ is the swaption prices that we can be observed in the market. By discritizing these integrals, one can easily calculated in the price of CMS rate.

1.3.2 Basis volatility

Now we will face the case basis risk, then the swap rate is given by

$$\mathrm{Swap} = \frac{\sum_i^n \delta_i\, \mathrm{FRA}_i \times \mathrm{DF}_i}{LVL}$$

The value of FRA is calculated from the Libor curve and the value of Level is computed from the discounting curve. Each bank has its own discounting curves, that means different banks may give different value of level. Indeed, the discounting curve is the same as the funding curve, then the unit asset which is calculated from the funding curve should be considered as risk-less for the bank. Then we define the cash swap rate which is computed from the discounting curve

$$S^c(t,T_0,T_n) = \frac{\mathrm{DF}(t,T) - \mathrm{DF}(t,T_n)}{\mathrm{LVL}(t,T_0,T_n)}.$$

And the basis $b(t,T_0,T_n)$ is given by

$$b(t,T_0,T_n) = S^c(t,T_0,T_n) - S(t,T_0,T_n).$$

We use the same argument as before

$$\mathrm{CMS}(0,T_0,T_n) = \mathrm{E}^{T_0}(S) = \mathrm{E}^{\,\mathrm{LVL}}\big[SM(S)\big].$$

And the annuity mapping is

$$M(S) = \mathrm{E}^{\,\mathrm{LVL}}\big[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}|S\big].$$

Let

$$M'(x) = \mathrm{E}^{\,\mathrm{LVL}}[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}|S^c],$$

be the the single (discounting curve) curve annuity mapping function. We consider first the case where the spread is deterministic, then it's not difficult to observe that :

$$M(x) = M'(x-b).$$

Or :

$$\mathrm{CMS} = \mathrm{E}^{\,\mathrm{LVL}}[\frac{S}{\alpha\ \mathrm{CLVL}(S^c)}] = \mathrm{E}^{\,\mathrm{LVL}}[\frac{S}{\alpha\ \mathrm{CLVL}(S+b_0)}],$$

where

$$\alpha = \mathrm{E}^{\,\mathrm{LVL}}[\frac{1}{\mathrm{CLVL}(S^c)}]$$

In practice, this is likely adequate for day-to-day risk management, since CMS risk usually sits far out on the curve (long expiries and long tenors) where the basis is relative stable. To study the impact of a stochastic basis, we assume that given $S^c(T_0)$, knowledge of $S(T_0)$ gives no additional information about the funding curve, then

$$\begin{aligned}\mathrm{E}^{\,\mathrm{LVL}}\big[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}|S\big] &= \mathrm{E}^{\,\mathrm{LVL}}\big[\mathrm{E}^{\,\mathrm{LVL}}[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}|(S,S^c)]|S\big] \\ &\approx \mathrm{E}^{\,\mathrm{LVL}}\big[\mathrm{E}^{\,\mathrm{LVL}}[\frac{\mathrm{LVL}_0}{\mathrm{LVL}(T_0,T_0,T_n)}|(S^c)]|S\big] \\ &= \mathrm{E}^{\,\mathrm{LVL}}\big[M'(S^c)|S\big].\end{aligned}$$

Noting that the mapping M' has the following form

$$M'(x) = \frac{1}{\alpha\ \mathrm{CLVL}(x)}.$$

For example in the case annual payment, we have

$$M'(x) = \frac{x}{\alpha(1-(1+x)^{-n})}.$$

Numerically we can assume that the function $1/\ \mathrm{CLVL}(x)$ is close to linear for the common value range of cash swap rate as in the Figure 5.1. We will study the impact of this assumption in the next chapter.

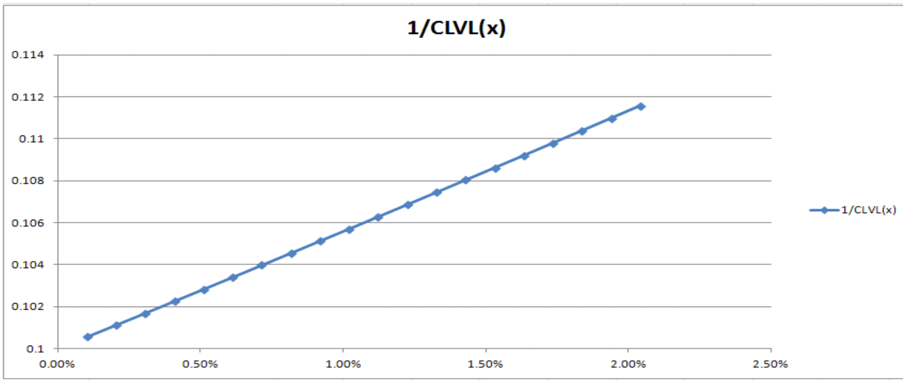

Figure 1.1: Function 1/CLVL(x) associated with a swap 10Y and x = 0% to 2%

Then we can move the expectation inside

$$M(S) \approx M'\big(\mathrm{E}^{\,\mathrm{LVL}}[S^c|S]\big) = M'\big(\mathrm{E}^{\,\mathrm{LVL}}[b|S] + S\big)$$

So the main question is how we project the cash rate or the basis spread onto swap rate ? In the special case where (S, b) is a gaussian vector with correlation ρ and σ_1, σ_2 is the ATM normal volatility implied of swap rate and cash rate respectively. Then we can obtain the projection of cash rate on Libor rate as a linear regression

$$\mathrm{E}^{\,\mathrm{LVL}}[S^c|S] = S + b_0 + \rho_{b,S}\frac{\sigma_b}{\sigma_S}(S - S_0)$$

where

- $\rho_{b,S}$ is the correlation between spread and Libor swap rate.

- σ_b, σ_S are are the ATM implied volatility of the Basis and Libor swap rate. And the CMS convexity adjustment is given by :

$$\boxed{\mathrm{CMS} \approx \mathrm{E}^{\,\mathrm{LVL}}\big(S \times \frac{1}{\alpha\ \mathrm{CLVL}(S + b_0 + \rho_{b,S}\frac{\sigma_b}{\sigma_S}(S - S_0)}\big)}$$

This formula shows that if the basis volatility is small and the correlation is small too, then the results must be the same as in the case deterministic spread.

Chapter 2

Numerical Tests

2.1 Overview

In finance, a pertinent model is a model which is coherent at least with the market and the results in classic case. Moreover, it should be simple and easy to compute and control the risk. In this chapter, we will try to verify the pertinent of this model and study the impact of basis volatility in this model.

Our main currency is Euro and the underlying are Libor swap rate 10Y and Basis (comparer to Cash swap rate 10Y). In this text we will consider only a market where the Libor swap volatility surface is gaussian, same as for Basis.

2.2 Verify models

In the first time, the two underlyings Libor swap rate and Basis have normal distribution. In this case, our goal is to verify if the correlation is zero and the basis have small volatility then the result should be the same as in the case deterministic spread.

Let's fix the basis volatility at 50 basic points and the correlation at 0%, then by using our numeraire change we obtain

Correl	0		
Basis vol	0.5		
Maturity	Copula	New Projection	Standard
5	20.42	20.45	20.34
10	33.94	34.02	33.43
15	36.07	36.19	35.90
20	38.94	39.12	41.56

Figure 2.1: Level of convexity adjustment in basic point

The standard output in the Figure (6.1) is given by the function CMSTEC, which calculate the CMS convexity adjustment in the case deterministic spread. And the copula price is given by using the copula method, in which we suppose that the copula model is Gaussian. This choice of copula as benchmark is natural in the Gaussian framework.

As we can observe in the Figure 6.1, there is small impact of the basis on the level of CMS convexity adjustments, as we expected in the case decorrelated.

Recall that one of the mail assumption that we use to construct the projection numeraire is the function $1/\,\mathrm{CLVL}(x)$ is close to be linear for usual rang of swap rate. So naturally if the basis volatility or the correlation term is high, our numeraire could have many chance to be out of usual rang. Let's consider a case limit where the correlation term is 100% and the basis volatility is 50 basic points as in the figure (6.2).

Correl	100%		
Basis vol	0.5		
Maturity	Copula	New Projection	Error
1	5.30	5.42	2.27%
5	31.13	32.39	4.04%
10	52.91	55.38	4.66%
15	60.83	61.85	1.68%
20	69.23	69.81	0.84%

Figure 2.2: Level of convexity adjustment in basis point

Numerically, the error is quite small and acceptable maybe except for the 10Y. Indeed, by supposing that the function $1/\,\mathrm{CLVL}(S+b)$ is linear we actually neglect the variance of Libor swap and basis.

$$\frac{1}{\mathrm{CLVL}(S+b)} = a + b(S+b-S_0-b_0) + O((S+b-S_0-b_0)^2)$$

That means the error is proportional to $\mathrm{cov}(S,(S+b)^2)$, ie quadratic in basis volatility as one can see in the Figure 6.3. The swap 10Y10Y has a high volatility and maturity compare to other swaps, that is why we observe the maximum error at the maturity 10Y.

Maturity	Basis vol =0.5	Basis vol = 1	Ratio
1	0.03%	0.10%	3.75
5	0.12%	0.46%	3.95
10	0.23%	0.90%	3.96
15	0.35%	1.37%	3.96
20	0.47%	1.84%	3.96

Figure 2.3: Error in function of basis volatility

2.3 Impact of basis volatility in the Gaussian model

2.3.1 Estimation of impact

Let us say here, the two underlyings are Libor swap rate and basis, and we consider first te impact of basis volatility and correlation term on the convexity adjustment given by the projection numeraire.

$$\mathrm{CMS} = \mathrm{E}^{\,\mathrm{LVL}}[\frac{S}{\alpha\ \mathrm{CLVL}(S+b_0+\rho\frac{\sigma_1}{\sigma_2}(S-S_0))}]$$

where σ_1 ,σ_2 are the implied volatility of basis and Libor swap rate respectively, and ρ is the correlation term between these two underlyings.

As we showed before, the function $1/\,\mathrm{CLVL}(x)$ is close to be linear, let's say by using Taylor expansion:

$$\frac{1}{\mathrm{CLVL}(S+b_0+\rho\frac{\sigma_1}{\sigma_2}(S-S_0)))} = \frac{1}{\mathrm{CLVL}(S_0+b_0)} + (1+\rho\frac{\sigma_1}{\sigma_2})G'(S_0+b_0)(S-S_0)$$

Then the CMS convexity adjustment is given by:

$$\mathrm{CMS} = S + \frac{G'(S_0+b_0)}{\alpha}(1+\rho\frac{\sigma_1}{\sigma_2}).Var(S) = S + \frac{G'(S_0+b_0)}{\alpha}(1+\rho\frac{\sigma_1}{\sigma_2})\sigma_2^2 T$$

That means :

$$\mathrm{CMS} = S + \frac{G'(S_0+b_0)}{\alpha}(\sigma_2^2+\rho\sigma_1\sigma_2)T$$

We can see that convexity adjustment given by the projection numeraire is linear in basis volatility and correlation term. And we know that, the error between the projection numeraire and the copula nuneraire is quadratic in basis volatility then the real convexity adjustment calculated by copula method should be linear in basis volatility and correlation too.

Moreover, for fixed level of Libor swap volatility then the convexity adjustment is just proportional to the product of basis volatility and correlation term, that means we can use this product as our simple parameter to estimate and control the risk of this model:

$$\text{Risk} = \text{Correl} \times \text{Basis Volatility}$$

2.3.2 Numerical result

In order to study the impact of basis volatility, we fix the correlation term at 50% and vary the basis volatility from 10bp to 200 bp and calculate the convexity adjustment which is given by copula method.

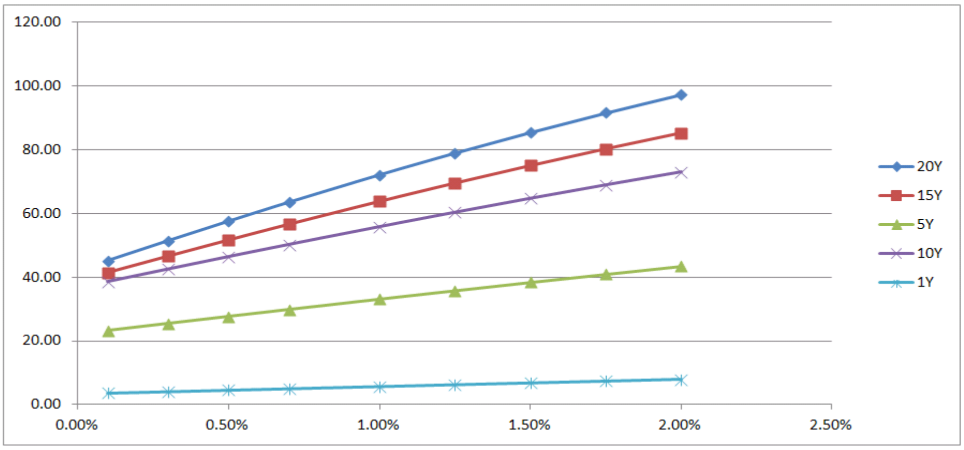

Figure 2.4: Convexity adjustment for various maturity

We observe well the linear dependence of convexity adjustment in function of basis volatility for various maturities. We can observe for the long maturity (for example for 10 maturity), the convexity adjustment varies from 40 basic points to 80 basis points, which is quite important.

The same kind of test for correlation term with fixed basis volatility at 50bp and correlation varies from 0% to 100%. And we also find that the convexity adjustment is linear in correlation term.

	Correlation							
Maturity	0%	10%	20%	30%	40%	50%	70%	100%
5Y	20.42	21.50	22.58	23.65	24.73	25.80	27.94	31.13
10Y	33.94	35.86	37.78	39.69	41.59	43.49	47.28	52.91
15Y	36.07	38.58	41.08	43.57	46.06	48.54	53.49	60.83
20Y	38.94	42.01	45.07	48.12	51.16	54.19	60.25	69.23

Figure 2.5: Fixed basis volitlity and study impact of correlation term

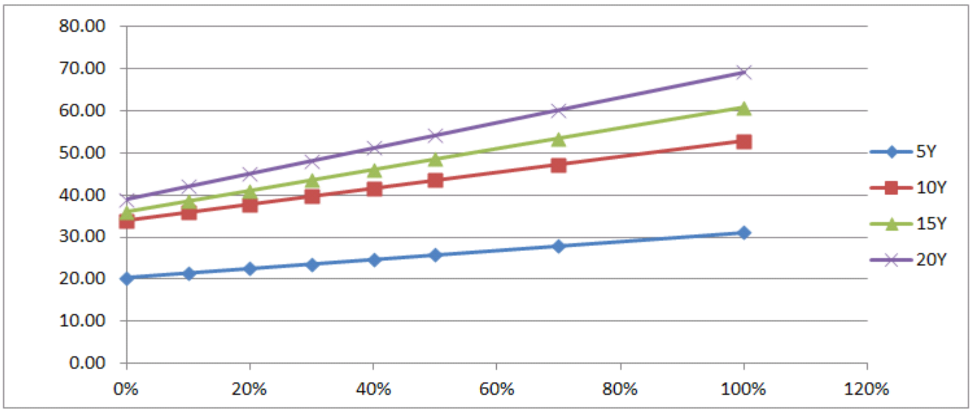

Figure 2.6: Convexity adjustment is linear in correlation term

From the figure 7.5, for example for the maturity 10Y, the level of convexity is almost double in the case full correlated compare to the case decorrelated. This fact proves that the basis risk is really important and we cannot ignore it in the pricing of CMS rate or CMS option.

2.4 Conclusion

In the case Libor swap rate and Basis have no smile, we can conclude that:

- In the pricing of CMS convexity adjustment, the impact of basis volatility and correlation term cannot be neglected, especially when the correlation term is not around zero.
- When the basis volatility and the correlation term are not too high (less than 150 basic points and 80% then the projection numeraire give us almost the same results as the true numeraire which is calculated by copula method.
- The error between the projection numeraire and the real numeraire is quadratic in basis volatility and important for the swap with long maturity and high volatility.
- The CMS convexity adjustment normally is linear in basis volatility and in correlation term.
- The risk term is simple and easily to calculate and control.