ABSTRACTS OF THE GEOMETRIC ASPECTS OF HODGE THEORY SESSION OF THE 2025 SRI IN ALGEBRAIC GEOMETRY

Yohan Brunebarbe. Constructing holomorphic functions on universal coverings of complex algebraic varieties.

Which complex analytic spaces can arise as the universal cover of a complex algebraic variety? Motivated by this question, Shafarevich asked whether the universal cover of a smooth projective variety X is always holomorphically convex — that is, whether there exists a proper holomorphic map from the universal cover of X to a Stein space. This was established in the linear case — when the fundamental group of X admits an almost faithful linear representation — by Eyssidieux–Katzarkov–Pantev–Ramachandran, using tools from non-abelian Hodge theory. In this talk, I will discuss a generalization of Shafarevich's question to the case of non-compact algebraic varieties. This is joint work with Ben Bakker and Jacob Tsimerman.

Philip Engel. Boundedness theorems for abelian fibrations.

I will discuss a proof that the number of deformation classes of symplectic varieties of fixed dimension, admitting a Lagrangian fibration, is finite (and analogous boundedness results for some other fibered K-trivial varieties). Conditional on the generalized semiampleness conjecture, we deduce a bound on the number of deformation classes of hyperkähler varieties in a fixed dimension, whose second Betti number is at least 5. This is joint work with Filipazzi, Greer, Mauri, and Svaldi.

Laure Flapan. Extremal divisors on moduli spaces of K3 surfaces.

We study the cone of pseudoeffective divisors on moduli spaces of K3 surfaces (and more generally on orthogonal modular varieties). We give numerical criteria for when (the irreducible components of) a Noether–Lefschetz divisor on these moduli spaces is an extremal ray of the pseudoeffective cone and use this to exhibit many new extremal divisors. We also discuss the question of whether the pseudoeffective cone is generated by Noether–Lefschetz divisors. This is joint work with Ignacio Barros and Riccardo Zuffetti.

Phillip Griffiths. Normal functions and their invariants.

Normal functions and their infinitesimal invariants are basic Hodge theoretic invariants of a pair (X, Z) where X is a smooth projective variety and Z is an algebraic subvariety or algebraic cycle in X. They have both a structure and applications to examples, and in this talk we will give an exposition of some aspects of the theory with emphasis on the singularity structure and some open issues.

Matt Kerr. Normal functions and torsion loci.

In his final published mathematical work, Poincaré studied holomorphic sections of a Jacobian bundle associated to a pencil of algebraic curves. When these sections behave "normally" (with “admissible” logarithmic singularities) along the discriminant locus, he showed that they arise from a 1-cycle on the total space. Lefschetz’s subsequent discovery, that fibering out a rational $(1,1)$-class on a smooth projective surface produces such a “normal function”, thus verified an instance of the Hodge Conjecture in the days before the Hodge decomposition was known.

Normal functions are to families of algebraic cycles what period maps are to families of algebraic varieties: the basic Hodge-theoretic invariant. Informally, they are given by integrals of differential forms over non-closed chains instead of topological cycles, and satisfy inhomogeneous differential equations instead of homogeneous ones. The simplest examples are sections of families of elliptic curves, and I will spend some time discussing such an example. After that I will give a general picture of how they arise, what the main current problems about them are, and state a few recent results — which invoke connections to o-minimal theory and quantization problems.

Bruno Klingler. Special loci for local systems.

Given a local system on a complex algebraic variety, what are the subvarieties on which the monodromy drops? The talk will discuss these monodromy special loci, a natural generalisation of (the positive period dimension components of) the Hodge loci.

Mirko Mauri. Combinatorial decomposition theorem for Hitchin systems via zonotopes

Many moduli problems in geometry are governed by discrete or continuous invariants. When it comes to study their cohomology, cohomology of moduli spaces related by continuous modifications can be studied via deformation techniques. However, we generally lack tools for understanding how the cohomology behaves when the modification of the moduli problem depends on discrete invariants.

In the celebrated case of Hitchin systems, we propose a strategy which combines Hodge theory, singularity theory and lattice point count in special polytopes called graphical zonotopes. This is a joint work with Luca Migliorini and Roberto Pagaria.

Colleen Robles. Completions of period maps.

I will report on a program to construct completions of period maps from nilpotent orbits.

Christian Schnell. Lagrangian fibrations and the support theorem.

Consider a Lagrangian fibration on a holomorphic symplectic variety. The simplest compact example is an elliptic K3 surface; the most interesting non-compact example is the Hitchin fibration. One can think of a Lagrangian fibration as being a special kind of “degenerating family of abelian varieties”, because the general fiber is an abelian variety. In a joint project with Mark de Cataldo and Yoonjoo Kim, we construct a group scheme that acts on the fibers of the Lagrangian fibration, and we use this action to prove that all Lagrangian fibrations satisfy Ngô’s support theorem (which controls the perverse sheaves that appear in the decomposition theorem).

Stefan Schreieder. Matroids and the integral Hodge conjecture for abelian varieties.

We prove that the cohomology class of any curve on a very general principally polarized abelian variety of dimension at least 4 is an even multiple of the minimal class. The same holds for the intermediate Jacobian of a very general cubic threefold. This disproves the integral Hodge conjecture for abelian varieties and shows that very general cubic threefolds are not stably rational. Our proof is combinatorial in nature and motivated by tropical geometry. Joint work with Phil Engel and Olivier de Gaay Fortman.

Sara Torelli. Holomorphic forms on curves and moduli.

The study of holomorphic forms on smooth projective curves and their moduli space conveys important geometric information, detecting, for instance, how the moduli space of smooth projective curves differs from its compactification. In this talk, I will present some results of non-extendability for holomorphic 1-forms on certain complete curves in the moduli space of smooth projective curves. In particular, this establishes that holomorphic 1-forms on the moduli space of smooth projective curves all vanish, as it was known to happen in their compactification since the 1970s. This is part of joint work with F. Favale and G.P. Pirola.

Qizheng Yin. Topology of abelian fibrations via algebraic cycles.

I will discuss some recent attempts to understand the topology of abelian fibrations by means of Fourier–Mukai duality and algebraic cycles. Issues we wish to address include the perverse filtration, the cohomology/Chow ring structure, the Chern classes, and the motivic nature of the decomposition theorem. I will present our general framework and expectations, and give a few examples where things are fairly well-understood. Based on joint projects with Younghan Bae, Davesh Maulik, and Junliang Shen.