Math 183
Statistical Methods

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Today: Chapter 2 (continued)

- Generalizing the "or" rule to non-disjoint events
- Joint, marginal and conditional probabilities
- Generalizing the "and" rule to non-independent events
- Reading these probabilities in contingency tables
- Encode natural language into probabilistic statements

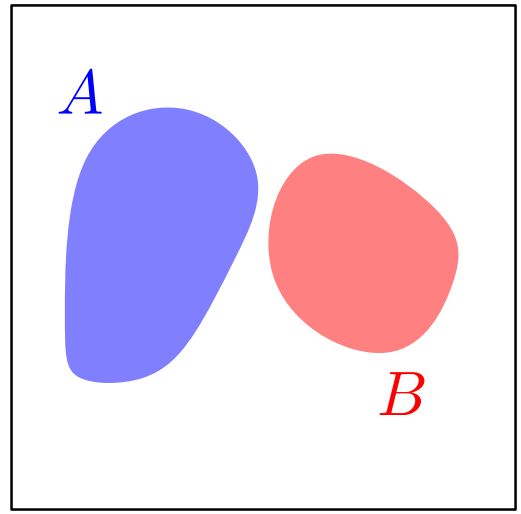
Recap of Last Lecture

- **"or" rule:** If two events A and B are disjoint,

$$P(A \text{ or } B) = P(A) + P(B).$$

Generalization: If $A_1, \ldots, A_n$ are disjoint,

$$P(A_1 \text{ or } \ldots \text{ or } A_n) = P(A_1) + \ldots + P(A_n).$$



- **"and" rule:** If two events A and B are independent,

$$P(A \text{ and } B) = P(A) \times P(B).$$

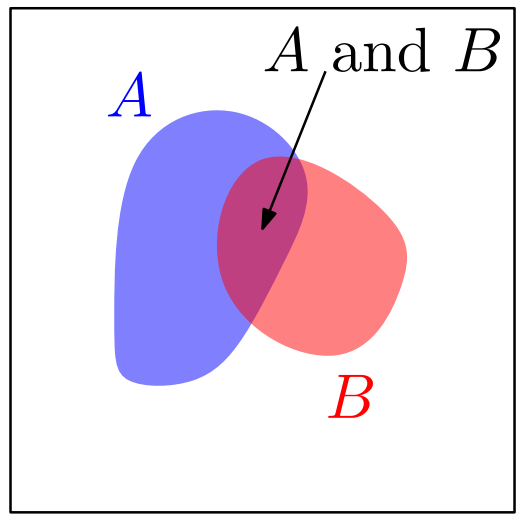
Generalization: If $A_1, \ldots, A_n$ are mutually independent,

$$P(A_1 \text{ and } \ldots \text{ and } A_n) = P(A_1) \times \ldots \times P(A_n).$$

"or" rule: Non-Disjoint Events

In general, for any two events A and B,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$



Caution: "or" is inclusive!

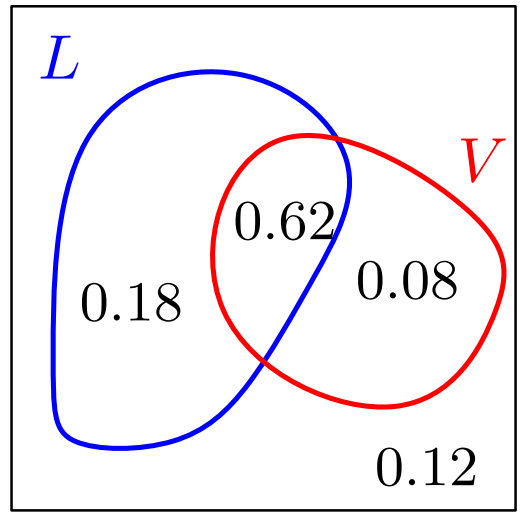
"A or B" = "A but not B, B but not A, A and B simultaneously".

Non-Disjoint Events: Example

80% of college students like learning. 70% of college student like video games. 62% like both learning and video games.
What percent like learning or video games?

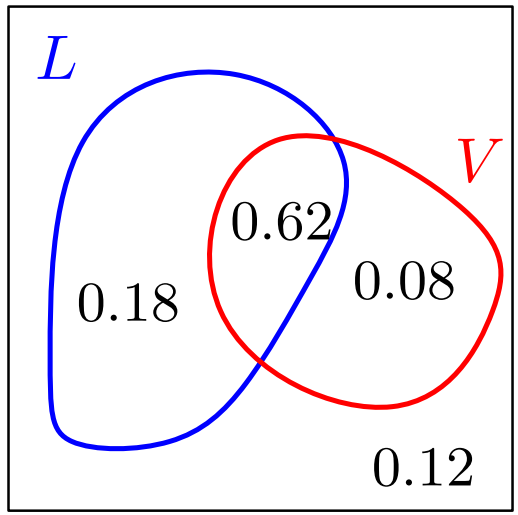
Let L be the event that a college student likes learning.
Let V be the event that a college student likes video games.

$$\begin{aligned} P(L \text{ or } V) &= P(L) + P(V) - P(L \text{ and } V) \\ &= 0.8 + 0.7 - 0.62 \\ &= 0.88. \end{aligned}$$



The picture on the right is called a **Venn diagram**.

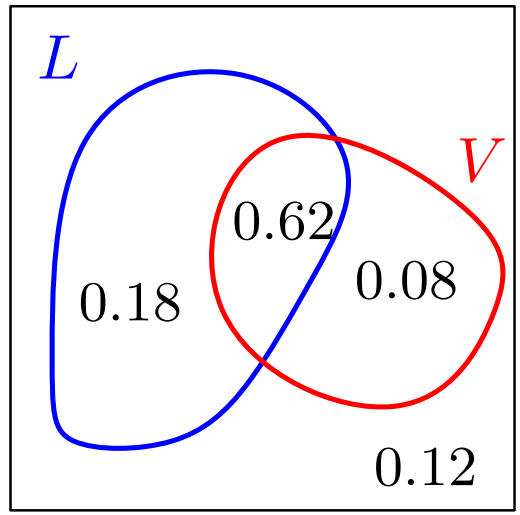
Non-Disjoint Events: Example



Describe in words the zone given by:

0.18	People who like learning but not video games.
0.08+0.62	People who like video games.
0.12	People who dislike learning and video games.
0.18+0.08	People who like learning only or video games only

Contingency Table



	Like video games	Dislike video games	Margin totals
Like Learning	0.62	0.18	0.8
Dislike Learning	0.08	0.12	0.2
Margin Totals	0.7	0.3	1

- **Joint Probabilities** are probabilities corresponding to two things happening simultaneously.
  Here: 0.62,0.18,0.08,0.12
- **Marginal Probabilities** are probabilities corresponding to the outcome of one variable.
  Here: 0.8,0.2 (for L) and 0.7, 0.3 (for V).

Contingency Table: Example

Among the students enrolled in the class,

- 41 are Junior with CS major
- 87 are neither Junior nor with CS major
- 131 have Major other than CS
- 107 are not Junior

What is the probability that a randomly chosen student is CS Major?

	Junior	Other Levels	Margin totals
CS Major	41	20	61
Other Major		87	131
Margin Totals		107	192

$$p = \frac{61}{192} \simeq 31.77\%$$

Losing Independence

Conditional Probability is a tool to handle events that are not independent.

Idea: When computing a probability, you actually have some extra information that you know is true.

Example:

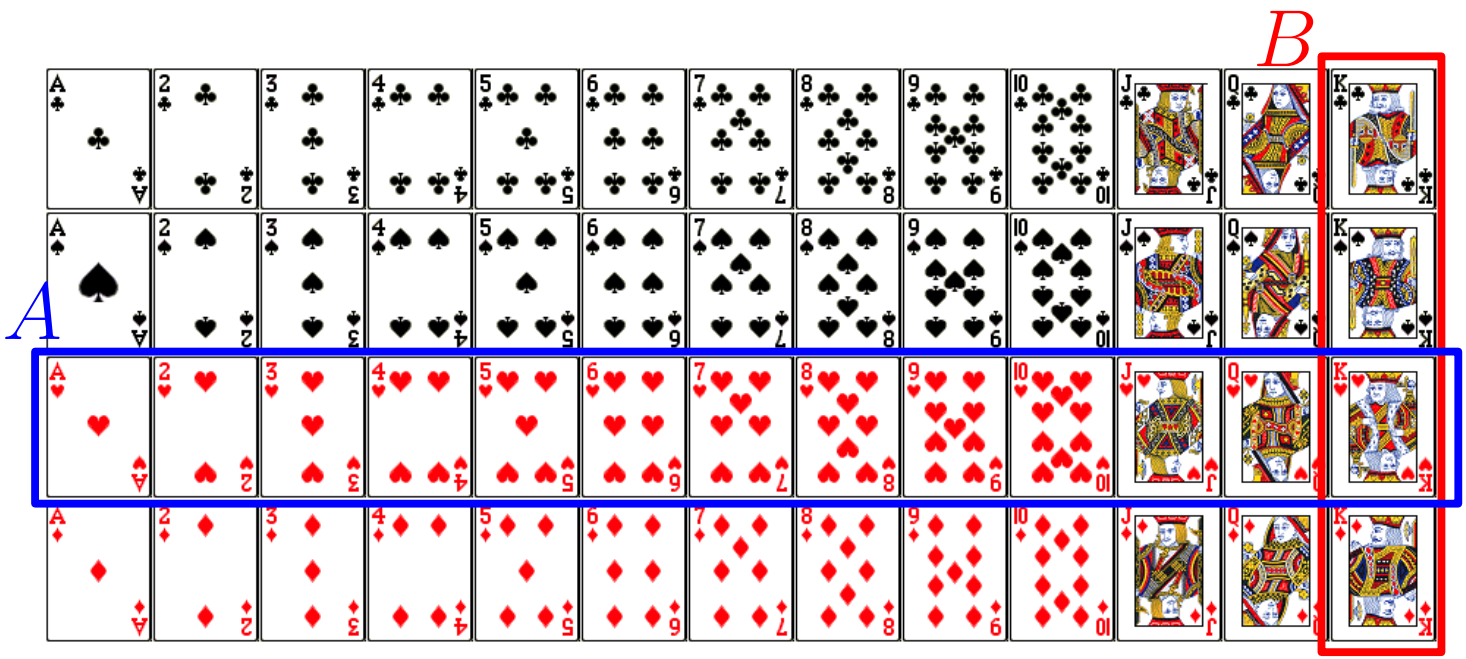
- $A = \{$ It will rain today in San Diego $\}$
- $B = \{$ You see dark storm clouds in the sky $\}$

Although $P(A) \simeq 11.2\%$ is small, you have a much higher chance to see A happen if you know already that B occurred.

Conditional Probability

A card is drawn from a deck. What is the probability that the card is a heart, given that the card is a king?

Intuition says 1/4.

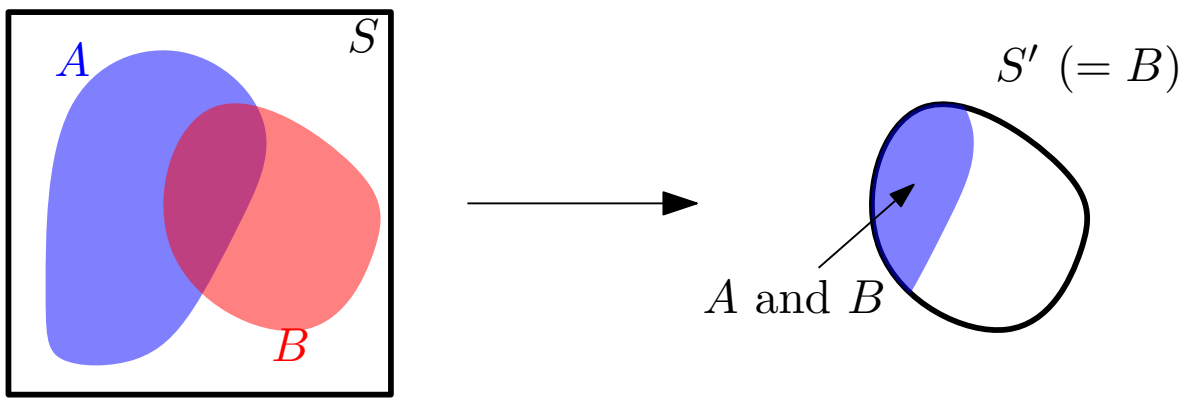


$$P(A \text{ given that } B \text{ occured}) = \frac{P(A \text{ and } B)}{P(B)}$$

Conditional Probability: Definition

For two event A, B the conditional probability of A given B is

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}.$$



Computing $P(A|B)$ amounts to do as if B was the sample space.

Notion of Independence Revisited

By definition, A and B are independent when

$$P(A \text{ and } B) = P(A) \times P(B).$$

But for any two events A and B, we have

$$P(A) = P(A|B) \times P(B).$$

Therefore,

$$\begin{aligned} A \text{ and } B \text{ are independent } &\Leftrightarrow P(A|B) = P(A) \\ &\Leftrightarrow P(B|A) = P(B). \end{aligned}$$

Checking for Independence Revisited

Independent or not?

1. Find $P(A)$
2. Find $P(A$ assuming you know event B has occured$) = \mathbf{P(A|B)}$

Do you get the same answer? $\mathbf{=}$ **Check if** $\mathbf{P(A|B) = P(A)}$

- **Yes:** Events are independent
- **No:** Events are NOT independent (= dependent)

Checking for Independence: Example

A poll led on randomlu picked North Carolina residents yields the following figures:

$P(\text{resident says gun ownership protects citizen}) = 0.58$

$P(\text{says guns protect citizens} \mid \text{is White}) = 0.67$

$P(\text{says guns protect citizens} \mid \text{is Black}) = 0.28$

$P(\text{says guns protect citizens} \mid \text{is Hispanic}) = 0.64$

The opinion on gun ownership varies by etchnicity, therefore the variables Opinion-on-guns and Ethnicity seem to be dependent.

Caution: Mind sample size!

Checking for Independence on Sample Data

- If conditional probabilities computed based on sample data suggest dependence between two variables, the next step is to conduct a **hypothesis test** to determine if the observed difference is **significant** or not
  (= to determine that this difference is likely to have been created by the sampling procedure or not)
- If the observed difference between the conditional probabilities is large, then there is stronger evidence that the difference is real.
- If the sample is large, then even a small difference can provide strong evidence of a real difference.

$\rightarrow$ See Chapter 4 later in the course.

Natural Language and Probabilities

```
> data("email")
> table(email$spam, email$number)
     none  small   big
  0   400   2659   495
  1   149    168    50
```

`email` contains data on 3921 emails sent to one user over 3 months. For each case:

- Write an expression for the given probability
- Say if this probability is marginal, joint, or conditional

- What percent of messages are spam with no number?
  $P($ spam and no number $)$, joint probability

$$P(\text{ spam and no number }) = \frac{149}{3921} \simeq 3.8\%$$

Natural Language and Probabilities

```
> data("email")
> table(email$spam, email$number)
    none small   big
0   400  2659   495
1   149   168    50
```

- What is the probability that a spam message will have a small number?
  $P($ small number $|$ spam $)$, conditional probability

$$P(\text{ small number } | \text{ spam }) = \frac{168}{149 + 168 + 50} \simeq 45.8\%$$

- What is the likelihood that a randomly-selected message with a big number will not be spam?
  $P($ not spam $|$ big number $)$, conditional probability

$$P(\text{ not spam } | \text{ big number }) = \frac{495}{495 + 50} \simeq 90.8\%$$

Natural Language and Probabilities

```
> data("email")
> table(email$spam, email$number)
    none small   big
0   400  2659   495
1   149   168    50
```

- What percent of messages do not contain a small number, ignoring the categorization of spam?
  $P($ not small $)$, marginal probability

$$P(\text{ not small }) = \frac{400 + 149 + 495 + 50}{3921} \simeq 27.9\%$$

- What fraction of emails would we expect to be non-spam with small or big numbers?
  $P($ not spam and not none $)$, joint probability

$$P(\text{ not spam and not none }) = \frac{2659 + 495}{3921} \simeq 80.4\%$$

What You Should do Now

- Turn in Homework 1!
- Start Homework 2 (due Friday, 13th 12:50pm)
- Finish reading Chapter 2.