## Homework 4 Math 181B, UCSD, Spring 2018 Due on Thursday, 10th May in class

Complete the following questions of the textbook from Larsen & Marx:

	$(5^{\text{th}} \text{ edition})$	
Section	Page	Questions
9.2	470	17, 18, 19, 20
9.3	475-476	6, 8, 9, 10
9.4	480	9, 10

In addition, complete the following exercises:

## Exercise 1

Consider the statistical model composed of distributions having densities

$$f_{\theta}(x) = \begin{cases} \theta^2 x \exp(-\theta x) & \text{if } x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

with respect to the Lebesgue measure, for  $\theta > 0$ .

Given  $X_1, \ldots, X_n \sim_{iid} f_{\theta}$ , study the consistency and the asymptotic normality of the maximum likelihood estimator of this model.

## Exercise 2

Recall that if  $X \sim Beta(\theta, 1)$  is Beta distributed for  $\theta \in (0, \infty)$ , it has density

$$f_{\theta}(x) = \begin{cases} \theta x^{\theta - 1} & \text{if } x \in (0, 1), \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that  $X_1, \ldots, X_n \sim_{iid} Beta(\mu, 1)$  and  $Y_1, \ldots, Y_m \sim_{iid} Beta(\nu, 1)$  are independent samples, with  $\mu, \nu > 0$  unknown.

- 1. Find the likelihood ratio statistic  $\Lambda$  for the test  $H_0$ :  $\theta = \mu$  against  $H_1$ :  $\theta \neq \mu$ .
- 2. What is the limiting distribution of  $-2\log\Lambda$ ?