Homework 6
Math 181B, UCSD, Spring 2018
Due on Thursday, 31st May in class

Complete the following questions of the textbook from Larsen & Marx:

(5^{th} edition)

Section	Page	Questions
14.3	668 and 676-677	1, 5, 6, 7, 8

In addition, complete the following exercises:

Exercise 1

We are interested in the failure rate of a data server. The server has been observed for $n = 100$ weeks. We denote by x_i the observed number of failures on week i. The obtained results are summarized in the following table:

# failures in week	0	1	2	3	≥ 4	Total
# weeks	35	34	23	6	2	100

Failures are assumed to be independent from each other.

1. At level $\alpha = 5\%$, can we conclude that the number of failures per week follows a Poisson distribution with parameter $\lambda = 1$?

2. At level $\alpha = 5\%$, can we conclude that the number of failures per week follows a Poisson distribution?

Exercise 2

Let $X_1, \ldots, X_n$ be i.i.d. real random variables with common density f and associated cumulative distribution function F. The Cramér-Von Mises is a classical alternative to Kolmorogov-Smirnov. It is based on the test statistic

$$I = \int_{\mathbb{R}} (F_n(x) - F(x))^2 f(x) dx,$$

where F_n denotes the empirical distribution function of the sample.
We let $(X_{(1)}, \ldots, X_{(n)})$ denote the sample sorted in increasing order (order statistic), so that $X_{(1)} = \min_{1 \leq i \leq n} X_i$ and $X_{(n)} = \max_{1 \leq i \leq n} X_i$.

1. Show that $F_n(x) = i/n$ if $X_{(i)} \leq x < X_{(i+1)}$ for $i \in \{1, \ldots, n-1\}$. What value takes $F_n(x)$ for $x < X_{(1)}$? What value takes $F_n(x)$ for $x \geq X_{(n)}$?

2. Show that

$$nI = \frac{1}{12n} + \sum_{i=1}^{n} \left(\frac{2i-1}{2n} - F(X_{(i)}) \right)^2 .$$

3. What is the distribution of the vector $(F(X_1), F(X_2), \ldots, F(X_n))$?

4. Show that the distribution of nI does not depend on the distribution of the sample.

(It is called the Cramér-Von Mises distribution of order n.)

5. Why is this property interesting to build a testing procedure?