PID:	
Last Name, First Name:	
Section:	
Approximate time spent to complete this assignment:	$\_$ hour(s)

# Homework 4 Math 11, UCSD, Winter 2018 Due on Tuesday, 13th February

Readings: Chapters 16.6-16.7 and the handout on continuous distributions.

#### Exercise 1

A large hospital has an average of 7 fatalities in a week. Using the Poisson model, what is the probability that this week it has 10 fatalities?

## Exercise 2

Suppose the probability of a major earthquake on a given day is 1 out of 10,000. What's the expected number of major earthquakes in the next 1000 days?

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I am the only bank teller on duty at my local bank. I need to run out for 10 minutes, but I don't want to miss any customers. Suppose the arrival of customers can be modeled by a Poisson distribution with mean 2 customers per hour.

1. What's the probability that no one will arrive in the next 10 minutes?

2. What's the probability that 2 or more people arrive in the next 10 minutes?

3. You've just served 2 customers who came in one after the other. Is this a better time to run out?

In an effort to improve the quality of their cell phones, a manufacturing manager records the number of faulty phones in each day's production run. The manager notices that the number of faulty cell phones in a production run of cell phones is usually small and that the quality of one day's run seems to have no bearing on the next day.

1. What model might you use to model the number of faulty cell phones produced in one day?

2. If the mean number of faulty cell phones is 2 per day, what is the probability that no faulty cell phones will be produced tomorrow?

3. If the mean number of faulty cell phones is 2 per day, what is the probability that 3 or more faulty cell phones were produced in today's run?

The cell phone manufacturer now wants to model the time between events. The mean number of defective cell phones is 2 per day.

4. What model would you use to model the time between events?

5. What would the probability be that the time to the next failure is 1 day or less?

6. What is the mean time between failures?

Almost every year, there is some incidence of volcanic activity on the island of Japan. In 2005 there were 5 volcanic episodes, defined as either eruptions or sizable seismic activity. Suppose the mean number of episodes is 2.4 per year. Let X be the number of episodes in the 2-year period 2010–2011.

1. What model might you use to model X?

2. What is the mean number of episodes in this period?

3. What is the probability that there will be no episodes in this period?

4. What is the probability that there are more than three episodes in this period?

Suppose X is a random variable with density  $f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$ 

1. Find  $P(X \le 1/2)$ .

2. Find  $P(X \ge 3/4)$ .

3. Find  $P(X \ge 2)$ .

4. Find E[X].

5. Find the standard deviation of X.

## Exercise 7

Suppose X is a random variable with density  $f(x) = \begin{cases} cx^2 & \text{if } 0 < x < 2\\ 0 & \text{otherwise} \end{cases}$  for some positive number c. What is c?



Suppose the number of years that a computer lasts has density  $f(x) = \begin{cases} 8x^{-3} & \text{if } x \ge 2\\ 0 & \text{otherwise.} \end{cases}$ 1. Find the probability that the computer lasts between 3 and 5 years.

2. Find the probability that the computer lasts at least 4 years.

3. Find the probability that the computer lasts less than 1 year.

4. Find the probability that the computer lasts exactly 2.48 years.

5. Find the expected value of the number of years that the computer lasts.

Suppose that the number of hours that it takes for a student to finish an exam has density

$$f(x) = \begin{cases} \frac{2}{5}(x+1) & \text{if } 1 < x < 2\\ 0 & \text{otherwise.} \end{cases}$$

1. Find the probability that the student finishes the exam in less than 1.5 hours.

2. Find the mean and standard deviation of the number of hours it takes to finish the exam.

Suppose that if you arrive at a bus stop at 8:00, the number of minutes that you will have to wait for the next bus is uniformly distributed on [0, 10].

1. Find the probability that you will have to wait at least six minutes for the bus.

2. Find the probability that you will have to wait between two and four minutes.

3. Find the expected value and standard deviation of the number of minutes that you have to wait for the next bus.

Suppose the random variable X has the uniform distribution on [a, b]. Find (by your own) expressions involving a and b for the expected value, variance, and standard deviation of X.

Suppose major earthquakes in a certain region occur independently of one another at the rate of one every ten years. Find the probability that the next major earthquake will occur between 7 and 12 years from now.

Suppose that the amount of time (in months) that a light bulb lasts before it burns out has an exponential distribution with parameter  $\lambda = 1/5$ .

1. What is the probability that it lasts at least three months?

2. If it has already lasted two months, what is the probability that it will last at least three more months?

3. On average, how many months will the light bulb last?

Accidents occur at a busy intersection at the rate of two per year. What is the probability that it will be at least one year before the next accident at the intersection? Compute the answer using the following two methods:

1. Let X be the number of accidents in the next year. Find the distribution of X and calculate P(X = 0).

2. Let T be the amount of time until the next accident. Find the distribution of T and calculate P(T > 1).