Math 11
Calculus-Based Introductory Probability and Statistics

Eddie Aamari
S.E.W. Assistant Professor

eaamari@ucsd.edu
math.ucsd.edu/~eaamari/
AP&M 5880A

Today:

- More hypothesis testing and confidence intervals
- Better understanding of p-values

Confidence Interval, Hypothesis Testing

Confidence Interval:

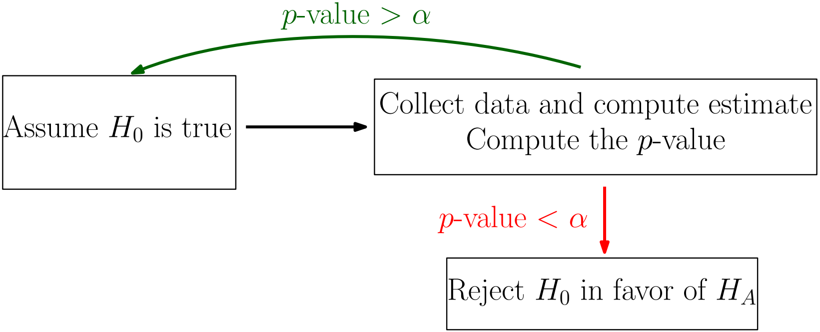
$$CI = \text{point estimate} \pm z^* \times SE_{model}.$$

Confidence Interval, Hypothesis Testing

Confidence Interval:

$$CI = \text{point estimate} \pm z^* \times SE_{model}.$$

Hypothesis Testing:



Another Example

You read that 5.6% of Americans identify as Asian. You wonder if San Diego is different. After sampling 400 random San Diegans, you find that 17 self-identify as Asian. What do you make of this?

Another Example

You read that 5.6% of Americans identify as Asian. You wonder if San Diego is different. After sampling 400 random San Diegans, you find that 17 self-identify as Asian. What do you make of this?

Let p be the percentage of Asians in San Diego.
(Always begin by defining your parameter)

Another Example

You read that 5.6% of Americans identify as Asian. You wonder if San Diego is different. After sampling 400 random San Diegans, you find that 17 self-identify as Asian. What do you make of this?

Let p be the percentage of Asians in San Diego.
(Always begin by defining your parameter)

H_0: $p = 0.056$
H_A: $p \neq 0.056$
(Choose one-sided vs. two-sided based on what would be interesting to you, not based on what the data suggest)

Another Example

You read that 5.6% of Americans identify as Asian. You wonder if San Diego is different. After sampling 400 random San Diegans, you find that 17 self-identify as Asian. What do you make of this?

Let p be the percentage of Asians in San Diego.
(Always begin by defining your parameter)

H_0: $p = 0.056$
H_A: $p \neq 0.056$
(Choose one-sided vs. two-sided based on what would be interesting to you, not based on what the data suggest)

We assume H_0 is true (to get started). In our particular sample, we get

$$\hat{p} = \frac{17}{400} = 0.0425.$$

By assuming H_0, we build a universe where $p = 0.056$, and any samples (of size $n = 400$) are drawn from such a universe.

By assuming H_0, we build a universe where $p = 0.056$, and any samples (of size $n = 400$) are drawn from such a universe.

The sampling distribution is approximately Normal if we meet the *Independence* and *10 Successes/Failures* conditions:

- We randomly chose people and 400 is far less than 10% of San Diego's total population.
- We expect $np = 400 \times 0.056 = 22.4 \geq 10$ successes and $nq = 400 \times 0.944 = 377.6 \geq 10$ failures.

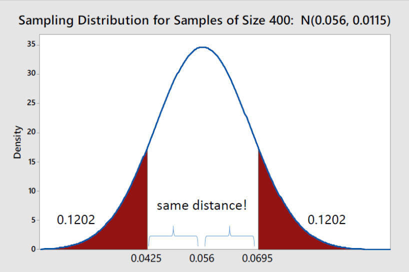
By assuming H_0, we build a universe where $p = 0.056$, and any samples (of size $n = 400$) are drawn from such a universe.

The sampling distribution is approximately Normal if we meet the *Independence* and *10 Successes/Failures* conditions:

- We randomly chose people and 400 is far less than 10% of San Diego's total population.
- We expect $np = 400 \times 0.056 = 22.4 \geq 10$ successes and $nq = 400 \times 0.944 = 377.6 \geq 10$ failures.

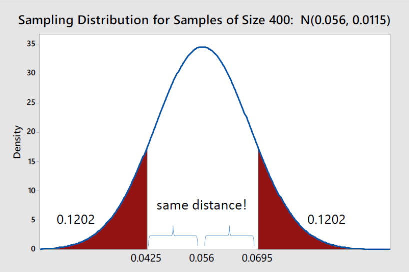
Sampling distribution: is (approximately) Normal with parameters

$$\mu = p = 0.056 \text{ and } SE = \sqrt{\frac{0.056 \times 0.944}{400}} \simeq 0.0115.$$



For a **two-sided alternative**, plot your sample and the symmetrically placed result in the picture (0.0425 and 0.0695).

Shade both tails.



For a **two-sided alternative**, plot your sample and the symmetrically placed result in the picture (0.0425 and 0.0695).

Shade both tails.

Our p-value is $2 \times 0.1202 \simeq 0.24 > 0.05$.
Here, we do not reject H_0.
Our result is not strange enough for us to abandon H_0.

Death Postponement

"Death Postponement": The theory that people will somehow delay their death until after an important life event (e.g., birthday, wedding of a child, etc...).

Death Postponement

"Death Postponement": The theory that people will somehow delay their death until after an important life event (e.g., birthday, wedding of a child, etc...).

Let p be the percentage of people that die in the three-month window before their birthdays.

Death Postponement

"Death Postponement": The theory that people will somehow delay their death until after an important life event (e.g., birthday, wedding of a child, etc...).

Let p be the percentage of people that die in the three-month window before their birthdays.

H_0: Death postponement is nonsense: $p = 1/4$.
H_A: Death postponement is real: $p < 1/4$.

Death Postponement

"Death Postponement": The theory that people will somehow delay their death until after an important life event (e.g., birthday, wedding of a child, etc...).

Let p be the percentage of people that die in the three-month window before their birthdays.

H_0: Death postponement is nonsense: $p = 1/4$.
H_A: Death postponement is real: $p < 1/4$.

Researchers looked at 747 deaths in Salt Lake City and found 60 deaths occurred in the three-month window before a person's birthday. *(Newsweek, 3/6/1978)*

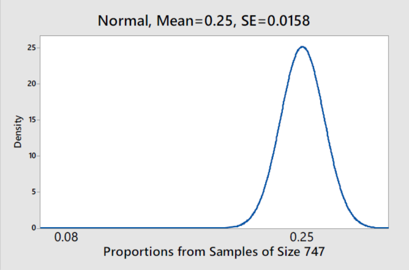
To the Sampling Distribution!

Assuming H_0, the universe should give us sample from

$$N\left(p, \sqrt{\frac{pq}{n}}\right) \simeq N\left(0.25, \sqrt{\frac{0.25 \times 0.75}{747}}\right)$$
$$= N(0.25, 0.0158)$$

Our data gives $\hat{p} = \frac{60}{747} \simeq 0.08$.

To the Sampling Distribution!



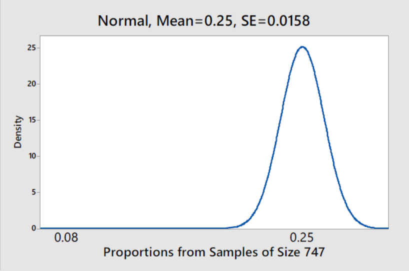
Assuming H_0, the universe should give us sample from

$$N\left(p, \sqrt{\frac{pq}{n}}\right) \simeq N\left(0.25, \sqrt{\frac{0.25 \times 0.75}{747}}\right)$$
$$= N(0.25, 0.0158)$$

Our data gives $\hat{p} = \frac{60}{747} \simeq 0.08$.

Shading the area to the left of 0.08 gives a p-value of $2.67 \cdot 10^{-27} \ll 0.05$.

To the Sampling Distribution!



Assuming H_0, the universe should give us sample from

$$N\left(p, \sqrt{\frac{pq}{n}}\right) \simeq N\left(0.25, \sqrt{\frac{0.25 \times 0.75}{747}}\right)$$
$$= N(0.25, 0.0158)$$

Our data gives $\hat{p} = \frac{60}{747} \simeq 0.08$.

Shading the area to the left of 0.08 gives a p-value of $2.67 \cdot 10^{-27} \ll 0.05$.

We reject H_0 in favor of H_A.

Where Does This 0.05 Cutoff Value Come From?

Where Does This 0.05 Cutoff Value Come From?

$\alpha = 0.05$ is a historical artifact derived from one sentence in a 1931 book by R.A. Fisher, *The design of Experiments*. He thought that a 1 in 20 event ($= 5\%$) might be surprising enough to toss out one's belief system (H_0) in favor of something else (H_A).

Where Does This 0.05 Cutoff Value Come From?

$\alpha = 0.05$ is a historical artifact derived from one sentence in a 1931 book by R.A. Fisher, *The design of Experiments*. He thought that a 1 in 20 event ($= 5\%$) might be surprising enough to toss out one's belief system (H_0) in favor of something else (H_A).

Some fields have a far more demanding threshold like $\alpha = 0.0000003$. This is usually called the "5 sigma rule": you need to see an event $5SE$'s from the assumend mean in order to discard H_0 in favor of H_A
Examples:

- Particule physics
- Pharmacology
- Aircraft design processes

P Overload!

- p is the proportion of some trait in a population.
  It is a parameter.
- $\hat{p}$ is the proportion of some trait in a sample.
  It is a statistic.
- $P(A)$ means the probability of some event A occuring.
  It is a probability.
- A p-value is a conditional probability:
  It is the probability of getting the value $\hat{p}$ (or something more extreme) in a universe where p is the law of the land. That is,

  $$p\text{-value} = P\left(\hat{p} \text{ or something more extreme} \mid H_0 \text{ is true}\right).$$

  It is calculated by finding an area under a sampling distribution curve, whose shape is determined by H_0.

Does Extra-Sensory Perception Exist?

In a 2011 article, Daryl Benn claims to have found evidence for Extra-Sensory Perception (ESP). Participants had to choose which of two curtains on a computer screen had an erotic picture behind it. They were able to do this 829 out of 1560 times.
Do these data suggest the ability to perceive erotica beyond what we expect from random chance?

Does Extra-Sensory Perception Exist?

In a 2011 article, Daryl Benn claims to have found evidence for Extra-Sensory Perception (ESP). Participants had to choose which of two curtains on a computer screen had an erotic picture behind it. They were able to do this 829 out of 1560 times.
Do these data suggest the ability to perceive erotica beyond what we expect from random chance?

H_0: ESP does not exist with erotic pictures.
H_A: ESP allows for better-than-random perception of erotic imagery.

Does Extra-Sensory Perception Exist?

In a 2011 article, Daryl Benn claims to have found evidence for Extra-Sensory Perception (ESP). Participants had to choose which of two curtains on a computer screen had an erotic picture behind it. They were able to do this 829 out of 1560 times.
Do these data suggest the ability to perceive erotica beyond what we expect from random chance?

H_0: ESP does not exist with erotic pictures.
H_A: ESP allows for better-than-random perception of erotic imagery.

Let p be the percentage of erotic pictures identified by those claiming to have ESP. We have

$$H_0\colon\ p = 0.5$$
$$H_A\colon\ p > 0.5$$

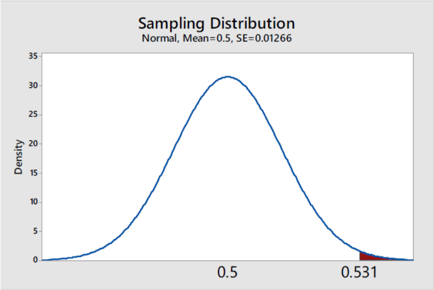
In this study, $\hat{p} = \dfrac{829}{1560} = 0.531.$

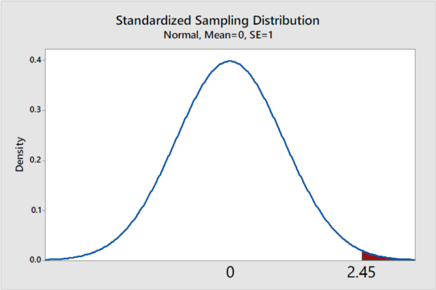
Under H_0, we are on the sampling distribution

$$N\left(0.5, \sqrt{\frac{0.5 \times 0.5}{1560}}\right) \simeq N(0.5, 0.01266).$$

Under H_0, we are on the sampling distribution

$$N\left(0.5, \sqrt{\frac{0.5 \times 0.5}{1560}}\right) \simeq N(0.5, 0.01266).$$





Tech approach: Use Minitab/calculator to find the P-value.

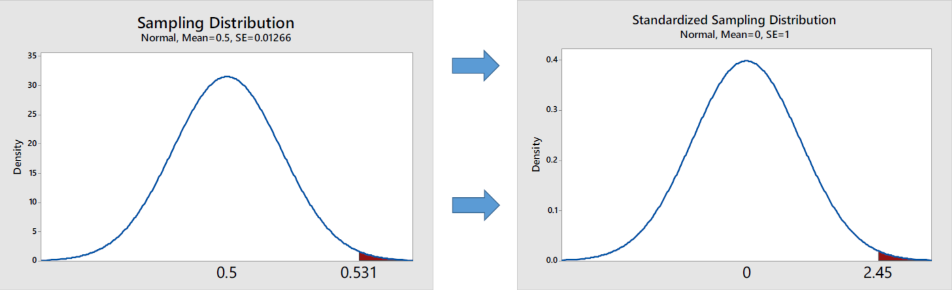
With $P = 0.007 < 0.05$, we reject H_0!

$$Z = \frac{0.531 - 0.5}{0.01266} \approx 2.45$$

Now use a Z-table to get the same answer.

Under H_0, we are on the sampling distribution

$$N\left(0.5, \sqrt{\frac{0.5 \times 0.5}{1560}}\right) \simeq N(0.5, 0.01266).$$



Tech approach: Use Minitab/calculator to find the P-value.

$$Z = \frac{0.531 - 0.5}{0.01266} \approx 2.45$$

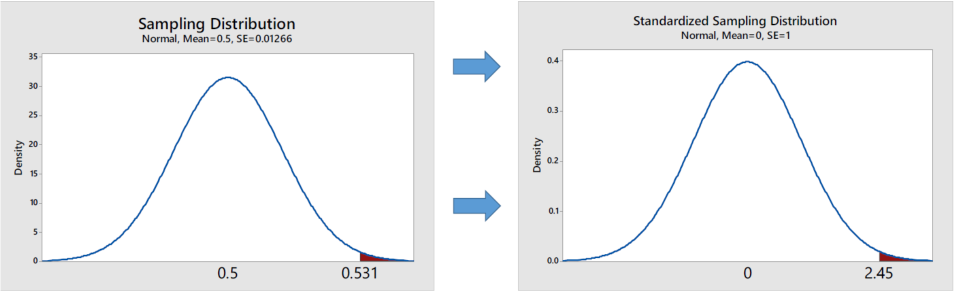
With $P = 0.007 < 0.05$, we reject H_0!

Now use a Z-table to get the same answer.

These data are strong enough to move to the alternative saying that ESP exists!!

Under H_0, we are on the sampling distribution

$$N\left(0.5, \sqrt{\frac{0.5 \times 0.5}{1560}}\right) \simeq N(0.5, 0.01266).$$



Tech approach: Use Minitab/calculator to find the P-value.

$$Z = \frac{0.531 - 0.5}{0.01266} \approx 2.45$$

With $P = 0.007 < 0.05$, we reject H_0!

Now use a Z-table to get the same answer.

These data are strong enough to move to the alternative saying that ESP exists!!

Such a study is part of the field of *Parapsychology*. For more info on such studies, see a conference of Chris French

Yet Another Example

Suppose that 62% of students who take the SAT eventually go on to college. You create an SAT prep class and enroll 500 random students on it. After the class, you discover 330 of your prep class kids go to college. Can you argue that your course causes a greater proportion of students to go to college?

Yet Another Example

Suppose that 62% of students who take the SAT eventually go on to college. You create an SAT prep class and enroll 500 random students on it. After the class, you discover 330 of your prep class kids go to college. Can you argue that your course causes a greater proportion of students to go to college?

Let p be the percentage of students who take your class that go on to college.

Yet Another Example

Suppose that 62% of students who take the SAT eventually go on to college. You create an SAT prep class and enroll 500 random students on it. After the class, you discover 330 of your prep class kids go to college. Can you argue that your course causes a greater proportion of students to go to college?

Let p be the percentage of students who take your class that go on to college.

Set H_0: $p = 0.62$ and H_A: $p > 0.62$.

Yet Another Example

Suppose that 62% of students who take the SAT eventually go on to college. You create an SAT prep class and enroll 500 random students on it. After the class, you discover 330 of your prep class kids go to college. Can you argue that your course causes a greater proportion of students to go to college?

Let p be the percentage of students who take your class that go on to college.

Set H_0: $p = 0.62$ and H_A: $p > 0.62$.

We have one sample with $\hat{p} = \frac{330}{500} = 0.66$.

Yet Another Example

Suppose that 62% of students who take the SAT eventually go on to college. You create an SAT prep class and enroll 500 random students on it. After the class, you discover 330 of your prep class kids go to college. Can you argue that your course causes a greater proportion of students to go to college?

Let p be the percentage of students who take your class that go on to college.

Set H_0: $p = 0.62$ and H_A: $p > 0.62$.

We have one sample with $\hat{p} = \dfrac{330}{500} = 0.66$.

How strange would this value be in a universe where $p = 0.62$?

Our universe will give rise to a sampling distribution which can be modelled by a Normal curve provided we meet the *Independence Assumption*, *10% Condition*, and *10 Successes/Failures Condition.*

Our universe will give rise to a sampling distribution which can be modelled by a Normal curve provided we meet the *Independence Assumption*, *10% Condition*, and *10 Successes/Failures Condition.*

Our students were chosen randomly, so they are independent of one another

In time, we will enroll many students in our prep course, so 500 students is less than 10% of our eventual population.

In a $p = 62\%$ universe, we expect $np = 500 \times 0.62 = 310$ successes and $nq = 500 \times 0.38 = 190$ failures, both of which are at least 10.

Our universe will give rise to a sampling distribution which can be modelled by a Normal curve provided we meet the *Independence Assumption*, *10% Condition*, and *10 Successes/Failures Condition.*

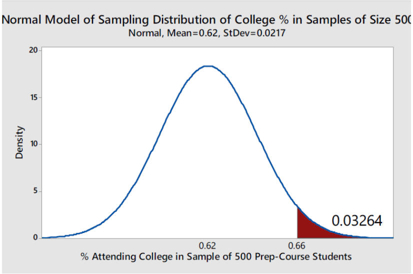
Our students were chosen randomly, so they are independent of one another

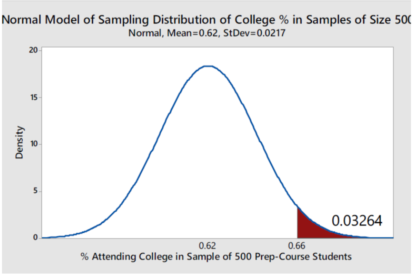
In time, we will enroll many students in our prep course, so 500 students is less than 10% of our eventual population.

In a $p = 62\%$ universe, we expect $np = 500 \times 0.62 = 310$ successes and $nq = 500 \times 0.38 = 190$ failures, both of which are at least 10.

We create a Normal model with

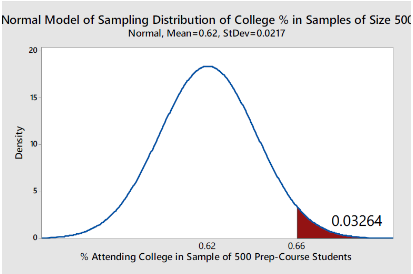
$$\mu = p = 0.62 \quad \text{and} \quad SE = \sqrt{\frac{0.62 \times 0.38}{500}} \simeq 0.0217.$$


Normal Model of Sampling Distribution of College % in Samples of Size 500
Normal, Mean=0.62, StDev=0.0217
Density
20
15
10
5
0
0.03264
0.62
0.66
% Attending College in Sample of 500 Prep-Course Students



What This *p*-Value Means:
Our p-value is about 0.033. This means that the probability of getting our 66% college attendance rate (or higher) is 0.033 (in a universe where our prep course is assumed to do nothing).

This is an unlikely event in our universe. Something is broken in our universe. The broken element is the assumption that the null hypothesis is true.



What This p-Value Means:
Our p-value is about 0.033. This means that the probability of getting our 66% college attendance rate (or higher) is 0.033 (in a universe where our prep course is assumed to do nothing).

This is an unlikely event in our universe. Something is broken in our universe. The broken element is the assumption that the null hypothesis is true.

Given that our p-value is $0.03264 < 0.05$, we reject the null hypothesis and accept the alternative hypothesis.

There is strong evidence that our prep course does make it more likely for students to attend college.

Summing Up Inference for Proportions

Summing Up Inference for Proportions

You want to explore a proportion p in some population. You begin by drawing a sample of size n and finding the statistic $\hat{p}$.

Summing Up Inference for Proportions

You want to explore a proportion p in some population. You begin by drawing a sample of size n and finding the statistic $\hat{p}$.

Common next steps:

- Build a confidence interval around $\hat{p}$
  - For 95% confidence or a difference level of confidence
  - Interpret this interval
  - Avoid common misconceptions about a confidence interval
- Conduct a hypothesis test to compare p with some reference value using your data $\hat{p}$
  - For $\alpha = 0.05$ or a different significance level
  - Interpret your p-value
  - Avoid common misconceptions about p-values.

Summing Up Inference for Proportions

You want to explore a proportion p in some population. You begin by drawing a sample of size n and finding the statistic $\hat{p}$.

Common next steps:

- Build a confidence interval around $\hat{p}$
  - For 95% confidence or a difference level of confidence
  - Interpret this interval
  - Avoid common misconceptions about a confidence interval
- Conduct a hypothesis test to compare p with some reference value using your data $\hat{p}$
  - For $\alpha = 0.05$ or a different significance level
  - Interpret your p-value
  - Avoid common misconceptions about p-values.

You should be able to do all these things with technology and with tables.

The Stock Market: Up or Down?

You are curious about what percent of days the Standard & Poor's Index gains money vs. loses money or is unchanged.

The Stock Market: Up or Down?

You are curious about what percent of days the Standard & Poor's Index gains money vs. loses money or is unchanged.

Let p be the percentage of days in the past when the index gained value.

The Stock Market: Up or Down?

You are curious about what percent of days the Standard & Poor's Index gains money vs. loses money or is unchanged.

Let p be the percentage of days in the past when the index gained value.

You randomly choose 500 days from the last 20 years and find 256 were up days for the S&P Index.

The Stock Market: Up or Down?

You are curious about what percent of days the Standard & Poor's Index gains money vs. loses money or is unchanged.

Let p be the percentage of days in the past when the index gained value.

You randomly choose 500 days from the last 20 years and find 256 were up days for the S&P Index.

Our sample gives us $\hat{p} = \dfrac{256}{500} = 51.2\%$.

The Stock Market: Up or Down?

You are curious about what percent of days the Standard & Poor's Index gains money vs. loses money or is unchanged.

Let p be the percentage of days in the past when the index gained value.

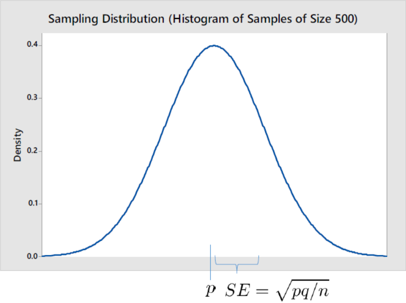
You randomly choose 500 days from the last 20 years and find 256 were up days for the S&P Index.

Our sample gives us $\hat{p} = \dfrac{256}{500} = 51.2\%$.

(What's wrong with claiming $p = 51.2\%$ or $\hat{p} \simeq 51.6\%$?)

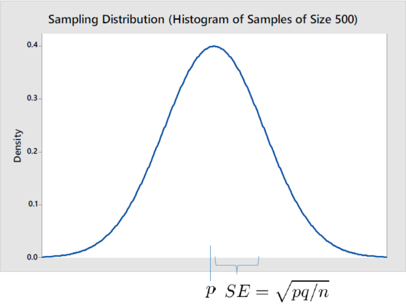
Constructing a CI

If the elements of the sample are independent and have at least 10 successes and failures, the sampling distribution is well modeled by $N(p, \sqrt{pq/n})$.



Constructing a CI

If the elements of the sample are independent and have at least 10 successes and failures, the sampling distribution is well modeled by $N(p, \sqrt{pq/n})$.



On any normal curve, 95% of the area is within 1.96 SDs of the center:

For 95% of the possible values of $\hat{p}$:

$p - 1.96 \cdot SE < \hat{p} < p + 1.96 \cdot SE$

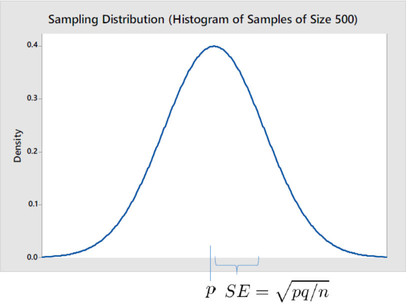
$\hat{p} - 1.96 \cdot SE < p < \hat{p} + 1.96 \cdot SE$

This is a CI!

This holds for 95% of the values for $\hat{p}$.

Constructing a CI

If the elements of the sample are independent and have at least 10 successes and failures, the sampling distribution is well modeled by $N(p, \sqrt{pq/n})$.



On any normal curve, 95% of the area is within 1.96 SDs of the center:

For 95% of the possible values of $\hat{p}$:

$p - 1.96 \cdot SE < \hat{p} < p + 1.96 \cdot SE$

$\hat{p} - 1.96 \cdot SE < p < \hat{p} + 1.96 \cdot SE$

This is a CI!

This holds for 95% of the values for $\hat{p}$.

Of all the CI's we could ever build taking $z^*1.96$ for critical value (centered at all the possible $\hat{p}$'s), 95% contain p.

95% Confidence and Other Levels

In the stock examples, $\hat{p} = 51.2\%$, so $SE = \sqrt{\dfrac{51.2 \times 48.8}{500}} \simeq 2.235\%$.

95% Confidence and Other Levels

In the stock examples, $\hat{p} = 51.2\%$, so $SE = \sqrt{\dfrac{51.2 \times 48.8}{500}} \simeq 2.235\%$.

So, our 95% CI is

$$\begin{aligned}\hat{p} \pm z^* \times SE &= 51.2 \pm 1.96 \times 2.235 \\ &= (46.82\%, 55.58\%).\end{aligned}$$

95% Confidence and Other Levels

In the stock examples, $\hat{p} = 51.2\%$, so $SE = \sqrt{\dfrac{51.2 \times 48.8}{500}} \simeq 2.235\%$.

So, our 95% CI is

$$\begin{aligned} \hat{p} \pm z^* \times SE &= 51.2 \pm 1.96 \times 2.235 \\ &= (46.82\%, 55.58\%). \end{aligned}$$

Suppose we want a 80% CI...

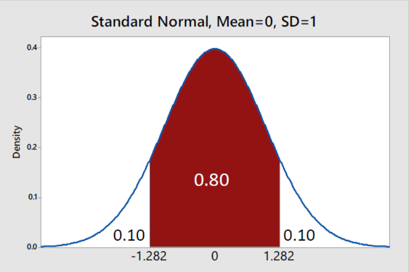
95% Confidence and Other Levels

In the stock examples, $\hat{p} = 51.2\%$, so $SE = \sqrt{\dfrac{51.2 \times 48.8}{500}} \simeq 2.235\%$.

So, our 95% CI is

$$\begin{aligned} \hat{p} \pm z^* \times SE &= 51.2 \pm 1.96 \times 2.235 \\ &= (46.82\%, 55.58\%). \end{aligned}$$

Suppose we want a 80% CI...



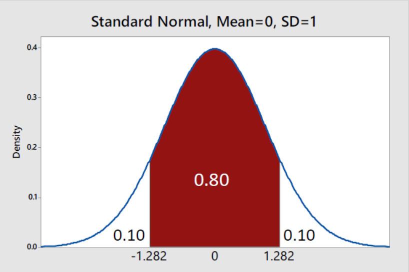
95% Confidence and Other Levels

In the stock examples, $\hat{p} = 51.2\%$, so $SE = \sqrt{\frac{51.2 \times 48.8}{500}} \simeq 2.235\%$.

So, our 95% CI is

$$\hat{p} \pm z^* \times SE = 51.2 \pm 1.96 \times 2.235$$
$$= (46.82\%, 55.58\%).$$

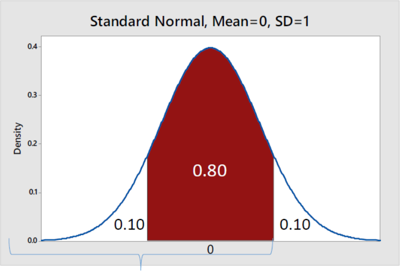
Suppose we want a 80% CI...



Our 80% CI is

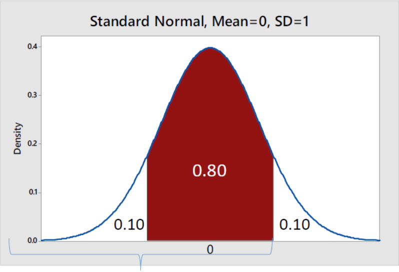
$$51.2 \pm 1.282 \times 2.235 = (48.33\%, 54.07\%)$$

Using a z-Table



The Z-table demands the area start at –infinity. So, we must search for 0.90 (0.10 + 0.80) in the table, not 0.80.

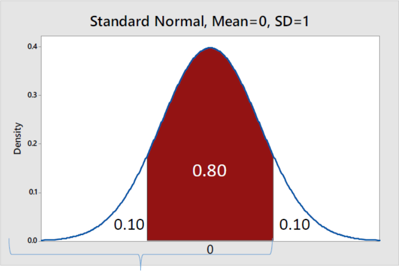
Using a z-Table



The Z-table demands the area start at –infinity. So, we must search for 0.90 (0.10 + 0.80) in the table, not 0.80.

Z	Second decimal place of Z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

Using a z-Table



The Z-table demands the area start at −infinity. So, we must search for 0.90 (0.10 + 0.80) in the table, not 0.80.

Z	Second decimal place of Z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	[illegible]	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	[illegible]	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	[illegible]	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	[illegible]	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	[illegible]	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	[illegible]	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	[illegible]	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	[illegible]	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	[illegible]	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	[illegible]	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	[illegible]	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	[illegible]	0.8830
1.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

So $z^* \simeq 1.28$. (Compare: 1.282 from technology)

CI Communication and Interpretation

Phrase-ology:

> *"We are 95% confident that the true proportion of up days for the S&P Index in the past is in* $(46.82\%, 55.58\%)$*."*

CI Communication and Interpretation

Phrase-ology:

> *"We are 95% confident that the true proportion of up days for the S&P Index in the past is in* $(46.82\%, 55.58\%)$*."*

95% confident: Of all the confidence intervals we might ever build (based on the different sample proportions we might get), 95% of the intervals will capture the true proportion.

CI Communication and Interpretation

Phrase-ology:

> *"We are 95% confident that the true proportion of up days for the S&P Index in the past is in* $(46.82\%, 55.58\%)$*."*

95% confident: Of all the confidence intervals we might ever build (based on the different sample proportions we might get), 95% of the intervals will capture the true proportion.

$(46.82\%, 55.58\%)$ These are the most plausible values for the population parameter, p, based on the data from our sample. The margin of error (about 4.38) shows how precise our interval is.
In this example, we have a fairly big window of possibilities for the population parameter.

CI Communication and Interpretation

Phrase-ology:

> *"We are 95% confident that the true proportion of up days for the S&P Index in the past is in* $(46.82\%, 55.58\%)$*."*

95% confident: Of all the confidence intervals we might ever build (based on the different sample proportions we might get), 95% of the intervals will capture the true proportion.

$(46.82\%, 55.58\%)$ These are the most plausible values for the population parameter, p, based on the data from our sample. The margin of error (about 4.38) shows how precise our interval is.
In this example, we have a fairly big window of possibilities for the population parameter.

There is evidence that many students and experts struggle to understand and interpret CIs! (Link)

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

The conditions necessary to do inference (build a CI) were met in this case

1. True
2. False

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

The conditions necessary to do inference (build a CI) were met in this case

1. True
2. False

Answer: True
Independence: From randomization and the <10% condition
10 S/F: here the sample proportion is 25%, so we had $64 \times 0.25 = 16$ successes and $64 - 16 = 48$ failures.

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

We are 95% confident that the percentage of these 64 people who are on government-subsidized health care is in $(22\%, 28\%)$.

1. True
2. False

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

We are 95% confident that the percentage of these 64 people who are on government-subsidized health care is in $(22\%, 28\%)$.

1. True
2. False

Answer: False
We are 100% sure the proportion about the 64 people is in $(22\%, 28\%)$.
Indeed, it is right in the middle, at 25%.
Statements about 95% are about the population **parameter**, not the sample statistic.

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

95% of random samples have a sample mean in the interval $(22\%, 28\%)$.

1. True
2. False

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

95% of random samples have a sample mean in the interval $(22\%, 28\%)$.

1. True
2. False

Answer: False
95% of sample means are within $2 \times SE$'s of the true parameter on the sampling distribution.
The given statement distorted the previous sentence.

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

A 99% confidence interval would be narrower than the 95% confidence interval since we need to be more sure of our estimate.

1. True
2. False

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

A 99% confidence interval would be narrower than the 95% confidence interval since we need to be more sure of our estimate.

1. True
2. False

Answer: False
The more you want to be sure the true parameter value lies in your CI, the broader it has to be.

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

The margin of error for our interval is 3%

1. True
2. False

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

The margin of error for our interval is 3%

1. True
2. False

Answer: True
$MOE = z^* \times SE$ (half the width of the CI)

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

There is a 95% chance that the population parameter is in our interval

1. True
2. False

Practice

A researcher is interested in calculating what percent of residents in her large city are currently on government-subsidized health care. She randomly draws a sample of 64 residents and calculates a 95% confidence interval to be $(22\%, 28\%)$.

There is a 95% chance that the population parameter is in our interval

1. True
2. False

Answer: False
The true population parameter is either **inside** or **outside** the CI. So roughly speaking, it's either 100% or 0%, but not 95%.

The Stock Market, Once More

Your friend claims that the stock market is just as likely to go up as it is to go down.
Does the sample you've drawn support this theory or not?

The Stock Market, Once More

Your friend claims that the stock market is just as likely to go up as it is to go down.
Does the sample you've drawn support this theory or not?

Recall: p is the percentage of days in the past when the index gained value.

H_0: $p = 50\%$ VS H_A: $p \neq 50\%$.
Notice that we choose a two-sided alternative based on what would be interesting, not based on any actual data.

The Stock Market, Once More

Your friend claims that the stock market is just as likely to go up as it is to go down.
Does the sample you've drawn support this theory or not?

Recall: p is the percentage of days in the past when the index gained value.

H_0: $p = 50\%$ VS H_A: $p \neq 50\%$.
Notice that we choose a two-sided alternative based on what would be interesting, not based on any actual data.

We assume H_0 is true (for now) and see what our earlier data ($\hat{p} = 51.2$) say about that assumption.

We still meet the conditions for inference, so our sampling distribution is:

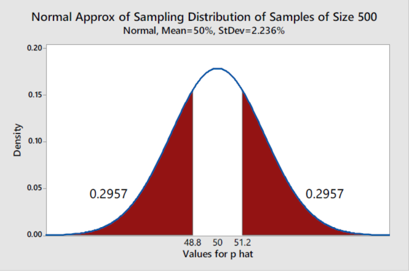
$$N\left(p, \sqrt{\frac{pq}{n}}\right) \simeq N\left(50, \sqrt{\frac{50 \times 50}{500}}\right) \simeq N(50, 2.236).$$

(This is a slightly different SE than earlier (for CIs) since we know (by assuming it!) what p is. Before we approximated p.)

We still meet the conditions for inference, so our sampling distribution is:

$$N\left(p, \sqrt{\frac{pq}{n}}\right) \simeq N\left(50, \sqrt{\frac{50 \times 50}{500}}\right) \simeq N(50, 2.236).$$

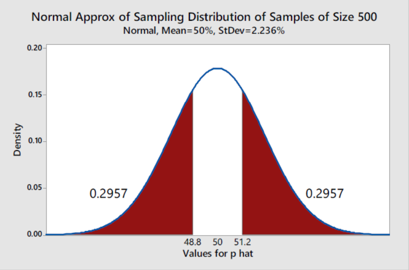
(This is a slightly different SE than earlier (for CIs) since we know (by assuming it!) what p is. Before we approximated p.)



We still meet the conditions for inference, so our sampling distribution is:

$$N\left(p, \sqrt{\frac{pq}{n}}\right) \simeq N\left(50, \sqrt{\frac{50 \times 50}{500}}\right) \simeq N(50, 2.236).$$

(This is a slightly different SE than earlier (for CIs) since we know (by assuming it!) what p is. Before we approximated p.)

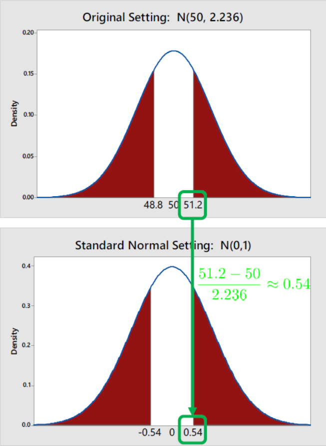


Our p-value is $2 \times 0.2957 \simeq 0.59$.

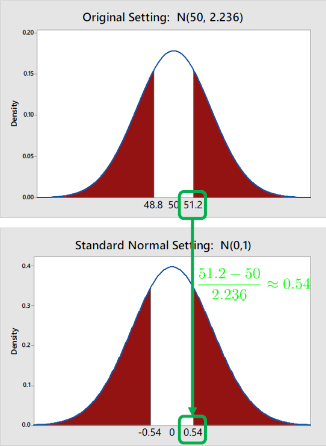
Since $0.59 > 0.05$, we do not reject H_0.

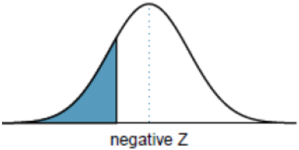
H_0 was a reasonable assumption when compared to the data $\hat{p} = 51.2\%$.

Using a z-Table



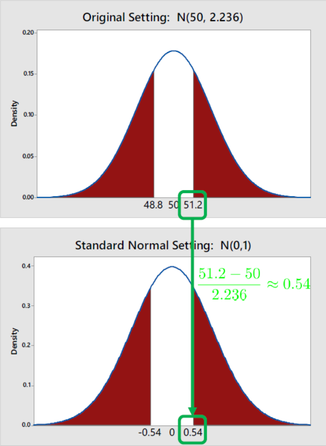
Using a z-Table

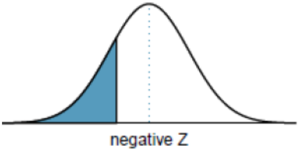




Second decimal place of Z							
0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

Using a z-Table





Second decimal place of Z							
0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

The z-table gives a p-value of $2 \times 0.2946 \simeq 0.59$.

Before, we had computers to find these areas, people convert their normal models to the standard normal model using z-scores.

Study $\hat{p}$ on $N(p, \sqrt{pq/n})$

How you can think when technology is available

Study $z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$ on $N(0, 1)$

How you must think when tables are used

Before, we had computers to find these areas, people convert their normal models to the standard normal model using z-scores.

Study $\hat{p}$ on $N(p, \sqrt{pq/n})$	Study $z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$ on $N(0,1)$
How you can think when technology is available	How you must think when tables are used

The act of forming a null hypothesis for a proportion, building a sample distribution, and using this to determine a p-value for an observed proportion $\hat{p}$ is known as a **one proportion z-test**.

p-Value Communication and Interpretation

There are many ways to communicate what a p-value means:

- **Definition**: $P(\text{data}^+|H_0) = 0.59$.
- **Setting**: If the stock market has 50% up days, the probability of seeing up-proportions like 51.2% (or more extreme) is about 0.5914.
- **Samples**: If the stock market has 50% up days, sampling variation would give results like 51.2% (or more extreme) in about 59 of 100 samples.
- **Strangeness**: If the stock market really has 50% up days, then we got a sample mean that is among the top 59% strangest sample means we could get.

p-Value Miscommunication and Misinterpretation

There is extensive evidence that students and researchers do not understand what p-values are. (Link)

A video in which statisticians struggle to explain a p-value: (Link)

In 2016, the American Statistical Society had to release an official statement on p-values because they are so misunderstood by everyone (students, researchers, statisticians, press, public) (Link)

Main Points:

- p-values can indicate how incompatible the data are with a specified statistical model.
- p-values do not measure the probability that the studied hypothesis is true.
- A p-value (statistical significance) does not measure the size of an effect or the importance of a result (practice significance).
- Scientific conclusions and business or policy decisions should not be based only on whether a p-value passes a specific threshold.