Math 11
Calculus-Based Introductory Probability and Statistics

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AP&M 5880A

Today:

- Two-Sample Means Inference

Statistics in the Large (Reloaded)

Where we stand: We know how to build C.I.'s and run hypothesis tests for one sample means.
So we can build a range of plausible values for a single parameter, or compare a single parameter to a known value.

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Something should sound different about these examples...

Are Your Two Populations Really Independent?

Two extremes:

- Knowing info about members of one population gives no helpful info about members in the other population
(Independent samples, 2-sample T-test)
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Pre-weight	Post-weight	Difference
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203	204	1
130	135	5
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Husband Age	Wife Age	Difference
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To analyze paired data, just do analysis on the differences!

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$$d = temp_{2016} - temp_{1970}$$

for each location, and find the differences d have $\bar{d} = 1.1°F$ with $s_d = 4.9°F$.

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Our differences will follow a T-distribution with $df = 62 - 1 = 61$.

We must calculate $\bar{d} \pm t^*_{61} \times SE_d$.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

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From the table, $t^*_{61} \simeq t^*_{60} \simeq 2.000$.

$$SE_{\bar{d}} = \frac{s_{\bar{d}}}{\sqrt{n}} = \frac{4.9}{\sqrt{62}} \simeq 0.622.$$

Thus, we have

$$\begin{aligned} &\bar{d} \pm t^*_{df} \times SE \\ &\simeq 1.1 \pm 2 \times 0.622 \\ &= (-0.144, 2.344). \end{aligned}$$

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3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

From the table, $t^*_{61} \simeq t^*_{60} \simeq 2.000$.

$$SE_{\bar{d}} = \frac{s_{\bar{d}}}{\sqrt{n}} = \frac{4.9}{\sqrt{62}} \simeq 0.622.$$

Thus, we have

$$\begin{aligned} \bar{d} \pm t^*_{df} \times SE \\ &\simeq 1.1 \pm 2 \times 0.622 \\ &= (-0.144, 2.344). \end{aligned}$$

We are 95% confident that temperature rose, on average (at the same location), between $-0.144°F$ and $2.344°F$ in the U.S. between Jan 1st, 1970 and Jan 1st, 2017.

Wait! What About the Conditions We Must Check?

Since two-sample paired data reduce to a 1-sample T-interval (or T-test) on the **differences**, we must simply check our usual conditions on the **differences** (which are the sample undergoing T-testing).

- Independence: The **differences** must be independent of one another. Since the differences are tied to the same location/person/couple, we just need those paired units to be independent of one another. This is usually checked via the Randomization Condition and the $< 10\%$ Condition.
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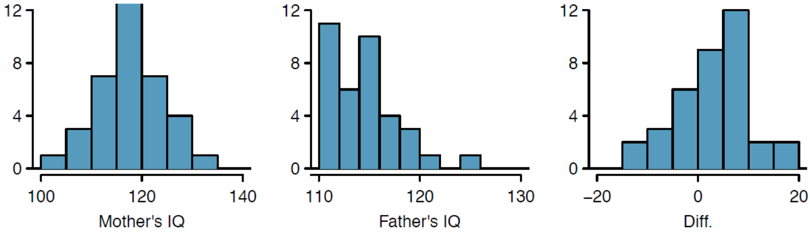
How paired units might fail to be independent of one another:
Choosing married couples that go to the same church, doing before/after experiments on college students, picking cities for the 1970/2016 temperature study that all fall in the same latitude, etc.

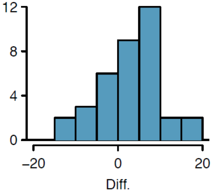
IQ of Parents of Gifted Children

Researchers collected IQ data on parents of 36 children identified as "gifted". Below are the results and histogram of the IQ differences of the parnts.
Run a test to see if mothers and fathers of gifted children have different average IQ's.



	Mother	Father	Diff.
Mean	118.2	114.8	3.4
SD	6.5	3.5	7.5
n	36	36	36

IQ of Parents of Gifted Children



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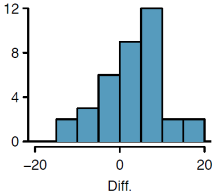

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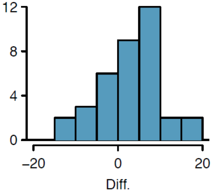

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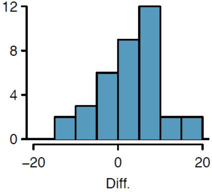

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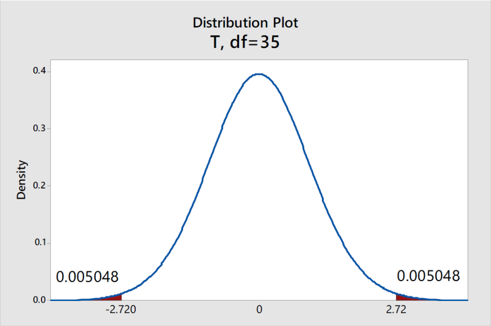
The t-score for our observed difference is

$$T = \frac{\bar{d} - 0}{SE} = \frac{3.4}{1.25} \simeq 2.72.$$

IQ of Parents of Gifted Children

If the alternative hypothesis is "H_A: $\mu_d \neq 0$", we find the following shaded area.



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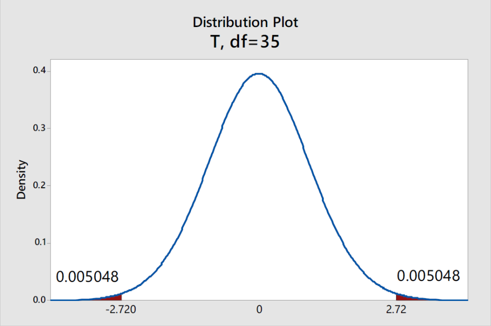

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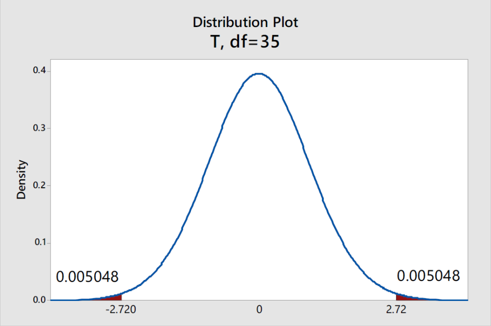

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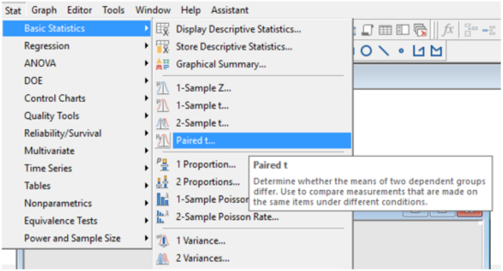
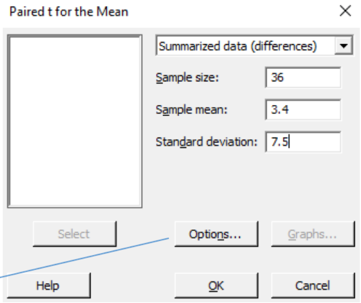
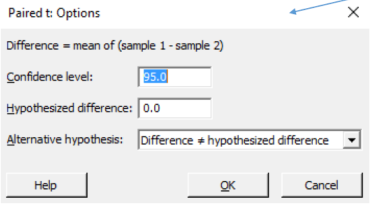
We reject the null hypothesis. It does appear that there is a difference in the average IQ's of parents of gifted children.







Paired T-Test and CI

```
             N   Mean   StDev   SE Mean
Difference  36   3.40    7.50      1.25


95% CI for mean difference: (0.86, 5.94)
T-Test of mean difference = 0 (vs ≠ 0): T-Value = 2.72  P-Value = 0.010
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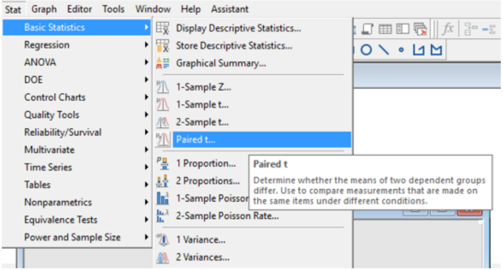

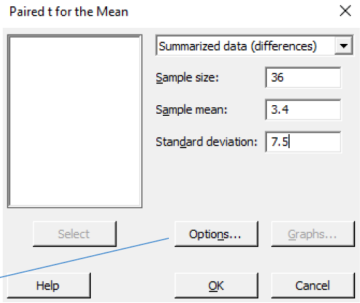

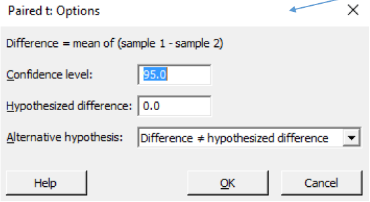

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Remember: Use Minitab to check, not to generate your answers.

Unpaired Independent Populations

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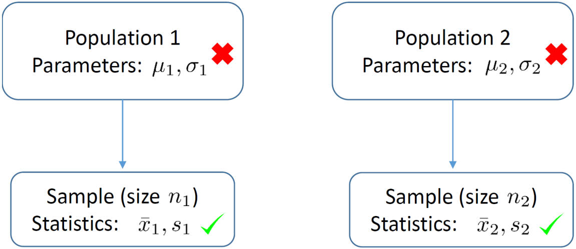

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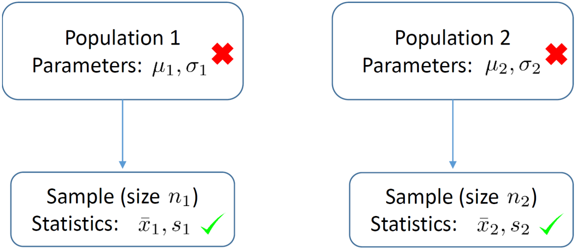

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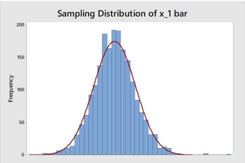
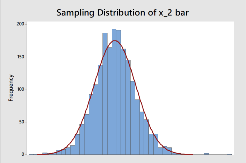


T-distribution with $df = n_1 - 1$ centered at μ_1 with $SE = \frac{s_1}{\sqrt{n_1}}$.

What does the sampling distribution of $\bar{x}_1 - \bar{x}_2$ look like?

Shape?
Center?
Spread?



T-distribution with $df = n_2 - 1$ centered at μ_2 with $SE = \frac{s_2}{\sqrt{n_2}}$.

Amazing Fact

If $\bar{X} = t_{n_1-1}$ and $\bar{Y} = t_{n_2-1}$ are independent random variables both modelled by T-distributions, then $\bar{X} - \bar{Y}$ is also a T-distribution with

$$df = \min(n_1 - 1, n_2 - 1).$$

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Furthermore, $\bar{X} - \bar{Y}$ is centered at

$$E(\bar{X} - \bar{Y}) = \mu_1 - \mu_2,$$

and has a SE which is found using the formula for the variance of a difference:

$$\begin{aligned} SE_{\bar{X}-\bar{Y}} = \sqrt{Var(\bar{X} - \bar{Y})} &= \sqrt{Var(\bar{X}) + Var(\bar{Y})} \\ &\simeq \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}. \end{aligned}$$

All C.I.'s Are Variations of One Another

C.I. for	Formula	SE	df
1 sample	$\bar{x} \pm t^*_{df} SE_{\bar{x}}$	$\frac{s}{\sqrt{n}}$	$n - 1$
2 paired samples	$\bar{d} \pm t^*_{df} SE_{\bar{d}}$	$\frac{s}{\sqrt{n}}$	$n - 1$
2 independent samples	$\bar{x}_1 - \bar{x}_2 \pm t^*_{df} SE_{\bar{x}_1 - \bar{x}_2}$	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$\min(n_1 - 1, n_2 - 1)$

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Keep in mind: When you change the scenario being discussed, you change the sampling distribution, and hence, the critical value and standard error.
To make C.I.'s of new ideas, we just need to know what the sampling distribution is and we are all set!

But Wait! Any Conditions For Us To Do Inference?

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To get each of the individual sampling distributions to be a t-distribution we need (in each sample):

- Independence of items in the sample (usually shown through the randomization and < 10
- Nearly normal population distribution (check via sample histogram where larger allows for skew)

To be able to subtract the t approximations for each sampling distribution and use our variance formula to get the SE of the difference:

- Independence of the two samples (no datum in one sample should help you predict any datum in the other sample)

Beetle Study (Again!)

We study beetle biodiversity in a pasture. For this, we collect a biodiversity index (Steinhaus index) in 2 different types of parcels:

1. in $n_1 = 12$ parcels where no animal grazes
2. in $n_2 = 13$ parcels with sheeps are grazing

Your get the following data:

$$\bar{x}_1 = 0.2505 \text{ and } s_1 = 0.0959$$

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Build a test with level of confidence $\alpha = 5\%$ to determine if animal grazing influences the biodiversity of beetles.

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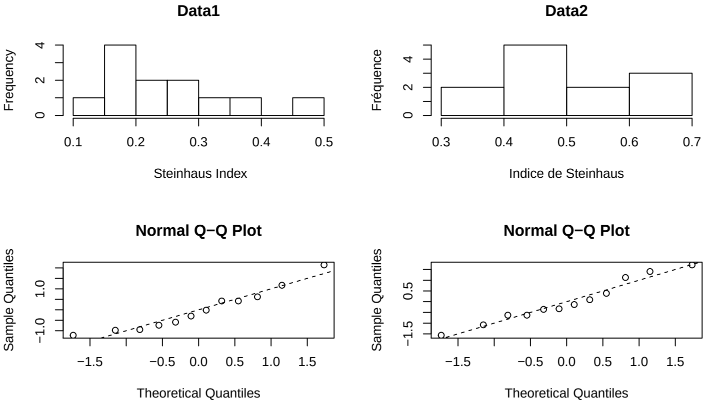

3) T-score your data

$$T = \frac{\text{point estimate} - \text{ null value}}{SE}$$

$$= \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{0.251 - 0.494}{\sqrt{\frac{0.095^2}{12} + \frac{0.107^2}{13}}} \simeq -6.011.$$

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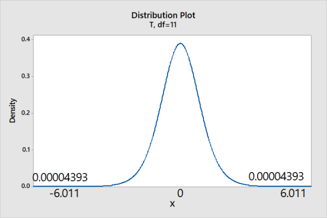

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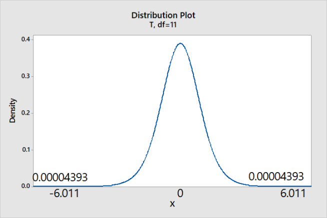

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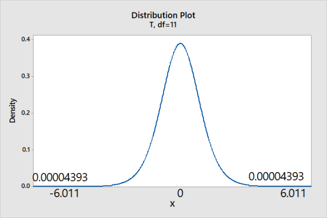

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Since $p \simeq 9 \cdot 10^{-5} \ll 0.05$, we reject H_0 and favor H_A.

There is (a very) strong evidence that the animal grazing influences beetle biodiversity.

A Big Helpful Chart

Things to Remember!	Difference in Proportions: Confidence Interval	Difference in Proportions: Hypothesis Test	Difference in Means: Confidence Interval	Difference in Means: Hypothesis Test
Independent samples (Chapter 22)	Sampling distribution of the differences is Normal! $SE = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$ Check that the samples are independent as well as the 2 conditions on each sample.	To have the best possible SE, we pool the data! Use $z = \frac{(\hat{p}_1 - \hat{p}_2) - (0)}{SE_{pooled}}$ where SE_{pooled} and $\hat{p}_{pooled}$ were defined last class. Check that the samples are independent as well as the 2 conditions on each sample.	Sampling distribution of the differences is a t-distribution with $df = \min(n_1 - 1, n_2 - 1)$ $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ Check that the samples are independent as well as the 2 conditions on each sample.	Use $t_{df} = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{SE}$ where $df = \min(n_1 - 1, n_2 - 1)$ No pooling is necessary. Check that the samples are independent as well as the 2 conditions on each sample.
Dependent samples that are paired (Chapter 23)	Not covered in Math 11	Not covered in Math 11	Do a 1-sample t-interval on the differences, d. Use $\bar{d} \pm t^*_{df} \cdot \frac{s_d}{\sqrt{n}}$ Check that the differences meet the 2 conditions.	Do a 1-sample t-test on the differences, d. Use $t_{df} = \frac{\bar{d} - 0}{SE}$ Check that the differences meet the 2 conditions.

A Few More Worked Examples

On any inference problem about means, you must do the following:

- Decide on the global setup (1 sample, 2 paired samples, 2 independent samples)
- Decide between CI or hypothesis test (one or two-sided)
- Check if conditions for inference are met
- Determine what sampling distribution we are on ($df = ?$)
- Find the SE and use it in the CI formula or to get the T-score

Technology and Food

Researchers wanted to see if using technology while eating would cause people to eat more food, perhaps because they were distracted. 44 patients were divided into equal treatment and control groups. The treatment group played computer Solitaire while eating; the control did not.

The weight (in grams) of food consumed were:

- treatment: mean 52.1, sd 45.1
- control: mean 27.1, sd 26.4

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Parameters: Let μ_T be the average weight of food of people eating while playing Solitaire (if the experiment were repeated for ever). μ_C is for control.

Hypotheses: $H_0 : \mu_T - \mu_C = 0$ and $H_A : \mu_T - \mu_C > 0$

(On this problem, it is tough to check the Nearly Normal condition in each sample. We likely meet the other conditions. We proceed cautiously.)

Sampling Distribution and Picture: For two means, our curve is a T-distribution with:

$$df = \min(22-1, 22-1) = 21, \text{ and } SE = \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}} \simeq 11.14.$$

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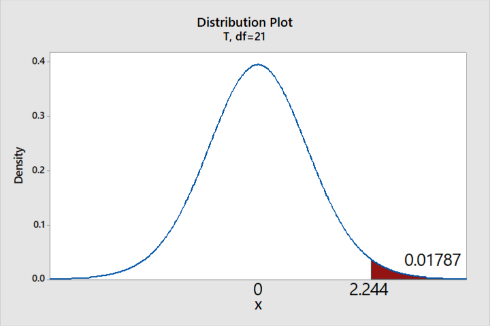

$$\begin{aligned} T &= \frac{(\bar{x}_T - \bar{x}_C) - 0}{SE} \\ &= \frac{52.1 - 27.1}{11.14} \\ &= 2.244. \end{aligned}$$

Can you find the P-value using a T-table?!

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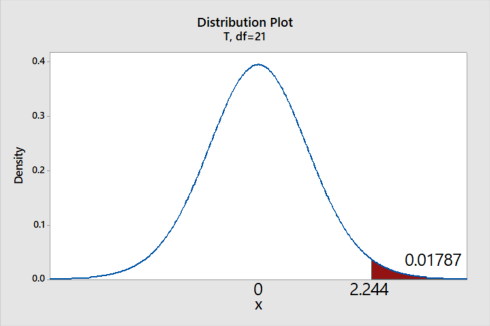

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Can you find the P-value using a T-table?!

Since $0.018 < 0.05$, reject H_0 in favor of H_A.
It does appear that distracted eating (via technology) leads to greater consumption.

Why Those Warning Labels on Cigarettes?

Researchers were interested if smoking was linked with lower birth weights of babies. They sampled 150 random North Carolina mothers and found the below data.

	smoker	non-smoker
mean weight (lbs)	6.78	7.18
st. dev.	1.43	1.60
sample size	50	100

Find a 90% confidence interval for $\mu_{non-smoke} - \mu_{smoke}$.

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The sampling distribution for the difference in the sample means is a T-distribution with $df = \min(50 - 1, 100 - 1) = 49$.

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one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	31	1.31	1.70	2.04	2.45	2.74
	32	1.31	1.69	2.04	2.45	2.74
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	34	1.31	1.69	2.03	2.44	2.73
	35	1.31	1.69	2.03	2.44	2.72
	36	1.31	1.69	2.03	2.43	2.72
	37	1.30	1.69	2.03	2.43	2.72
	38	1.30	1.69	2.02	2.43	2.71
	39	1.30	1.68	2.02	2.43	2.71
	40	1.30	1.68	2.02	2.42	2.70
	41	1.30	1.68	2.02	2.42	2.70
	42	1.30	1.68	2.02	2.42	2.70
	43	1.30	1.68	2.02	2.42	2.70
	44	1.30	1.68	2.02	2.41	2.69
	45	1.30	1.68	2.01	2.41	2.69
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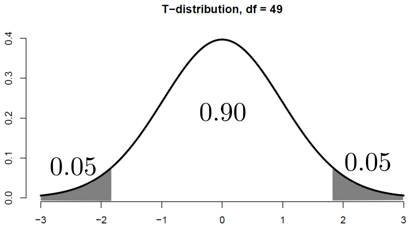

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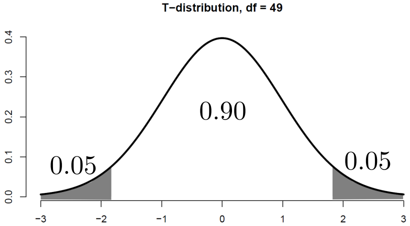

We find $t^*_{49} = 1.68$.

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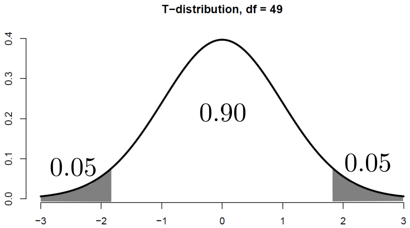

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Since $\bar{x}_1 - \bar{x}_2 = 7.18 - 6.78 = 0.4$, we have

$$CI = 0.4 \pm 1.68 \times 0.258 = (-0.03, 0.83).$$

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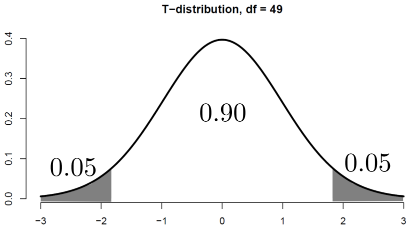

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$$CI = 0.4 \pm 1.68 \times 0.258 = (-0.03, 0.83).$$

We are 90% confident that babies born to non-smoking NC women are about 0.83 to -0.03 lbs heavier than babies born to smoking NC women.

Your Turn!

Which of the following scenarios involve paired data?

1. Comparing students' self-reports of "love for statistics" before and after E. Aamari's class.
2. Assessing the gender-related salary gap by comparing salaries of men and women in the same randomly sampled positions at the same companies.
3. Comparing lung capacity changes in athletes before and after six weeks of training.
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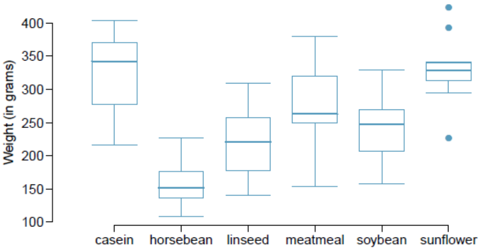
Answer:

1. Paired. The linkage in the student.
2. Paired. The linkage is the common job.
3. Paired. The linkage is the athlete.
4. No paired. Paired data would be people trying both.
5. Paired. The linkage is marriage.

Let's Get Huge!



	Mean	SD	n
casein	323.58	64.43	12
horsebean	160.20	38.63	10
linseed	218.75	52.24	12
meatmeal	276.91	64.90	11
soybean	246.43	54.13	14
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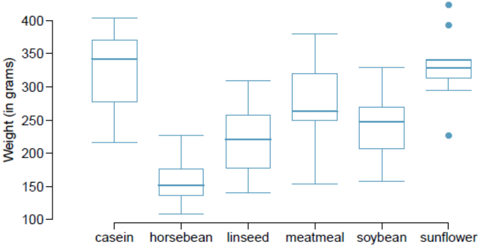

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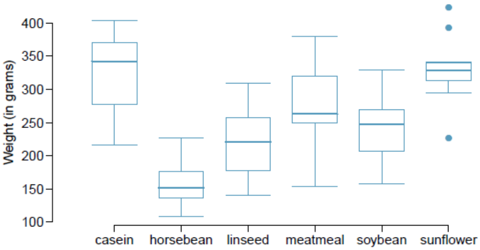
Remark: Given the small sample sizes and skew seen in the boxplot of meatmeal and casein, **we should not proceed with inference**. We probably don't meet the Nearly Normal condition needed for each sample.



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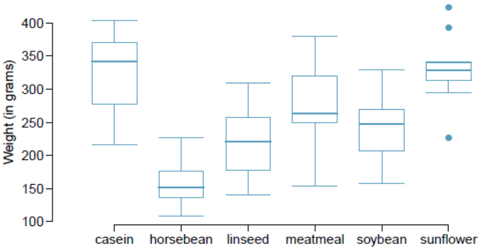

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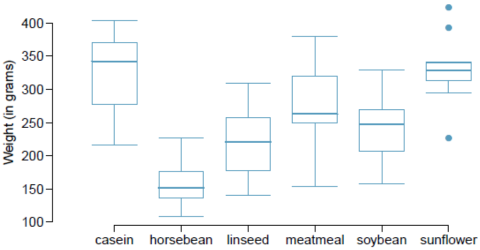
Our point estimate for the (unknown) parameters $\mu_L - \mu_H$ is

$$\bar{x}_L - \bar{x}_H = 218.75 - 160.20 = 58.55 \text{ grams.}$$



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Our sampling distribution is t_9 with

$$SE = \sqrt{\frac{52.24^2}{12} + \frac{38.63^2}{10}} \simeq 19.41.$$

From the table, $t_9^* = 2.262$, so

$$
\begin{aligned}
CI &= 58.55 \pm 2.262 \times 19.41 \\
&= (14.64, 102.46)
\end{aligned}
$$

Notice that 0 isn't in this interval. So the difference in parameter values is unlikely to be 0.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

From the table, $t_9^* = 2.262$, so

$$CI = 58.55 \pm 2.262 \times 19.41$$
$$= (14.64, 102.46)$$

Notice that 0 isn't in this interval. So the difference in parameter values is unlikely to be 0. Under

$H_0 : \mu_L - \mu_H = 0$, we get:

$$T = \frac{58.55 - 0}{19.4}$$
$$\simeq 3.018.$$

With a two-sided alternative, the p-value is $p = P(|T_9| > 3.018)$. From the table, p satisfies

$$0.01 \leq p \leq 0.02$$

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
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z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
					Confidence Level						

From the table, $t_9^* = 2.262$, so

$$\begin{aligned} CI &= 58.55 \pm 2.262 \times 19.41 \\ &= (14.64, 102.46) \end{aligned}$$

Notice that 0 isn't in this interval. So the difference in parameter values is unlikely to be 0. Under

$H_0 : \mu_L - \mu_H = 0$, we get:

$$\begin{aligned} T &= \frac{58.55 - 0}{19.4} \\ &\simeq 3.018. \end{aligned}$$

With a two-sided alternative, the p-value is $p = P(|T_9| > 3.018)$. From the table, p satisfies

$$0.01 \leq p \leq 0.02$$

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
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two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
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3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
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z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

The p-value $p \leq 0.02 < 0.05$ leads us to reject the null $H_0 : \mu_L - \mu_H = 0$ in favor of $H_A : \mu_L - \mu_H \neq 0$. (As already guessed with the CI)