Math 11
Calculus-Based Introductory Probability and Statistics

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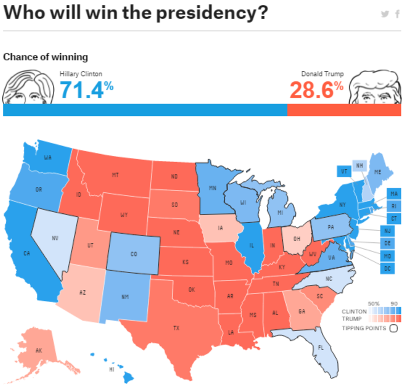
Today:

- Introduction to Probability Theory

Concepts of Probability

Formalizing everyday life's sentences:

- "You have a 30% chance of surviving this illness."
- There's a 50/50 chance of getting heads when you flip a fair coin."



The notion of **probability** covers several related insights.

Different Ways of Thinking About Probability

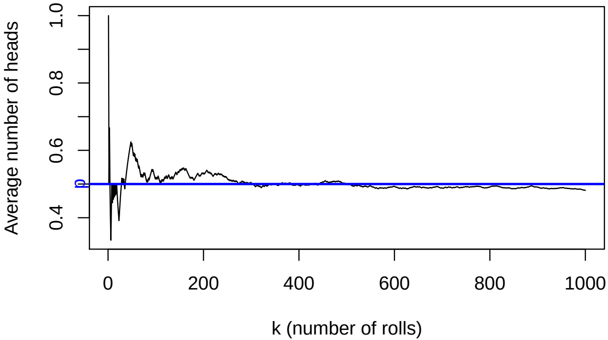
Bayesian point of view: A probability is a subjective degree of belief. For the same outcome, two persons could have different viewpoints and so assign different probabilities
Example: Sports supporters' groups would say their team is more likely to win.

Frequentist approach: The probability of an outcome is the proportion of times the outcome would occur if we observed the random process an infinite number of times.

Law of Large Numbers

The **Law of Large Numbers** (LLN) states the following:

As a random process is repeated more and more,
the proportion of times that an event occurs converges to a number
(the probability of that event!).



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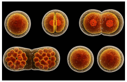
Cell division is random...



... While at the microscopic level, all the processes involved are deterministic (both chemical and physical ones).

What is Randomness?

It can be helpful to model a process as random even if it is not truly random. Random processes often model some lack of information.

Example: coin flip, stock markets, biological processes, ecology,...

Handling randomness is often much more convenient and informative than dozens/thousands parameters.

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Going further: introducing randomness artificially can even help in prospective studies
Example: Polls (=Picking people at random in the population and asking them their opinions)

Probability Vocabulary

Let us define some words to talk about theoretical probability:

- **Trial:** Action that creates the data
- **Outcome:** The data created
- **Event:** Some set of outcomes you might care about
- **Sample Space:** The set of all possible outcomes

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Let's illustrate this with **Coin flip**:

- **Trial:** Flipping the coin
- **Outcome:** Heads (say)
- **Event:** $A = \{\text{flipping Heads}\}$.
- **Sample Space:** $S = \{\text{Heads,Tails}\}$

Probability Vocabulary

With a **Deck of cards**:

- **Trial:** Drawing a card at random
- **Outcome:** Ace of hearts (say)
- **Event:** $B = \{\text{drawing a King}\}$.
- **Sample Space:** $S = \{\text{the 52 cards}\}$

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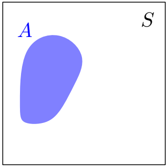
If you are **Gathering info on Facebook usage**:

- **Trial:** Asking three random people if they have a Facebook account
- **Outcome:** NYY (say)
- **Event:** $C = \{\text{all 3 say yes}\}$.
- **Sample Space:**
  $S = \{\text{YYY, YYN, YNY, NYY, YNN, NYN, NNY, NNN}\}$

Defining Probability

If all the outcomes in a (finite) sample space are equally likely, the probability of an event A is defined to be

$$P(A) = \frac{\#\text{ of outcomes in event } A}{\#\text{ of outcomes in sample space } S} = \frac{\#\text{ favorable outcomes}}{\#\text{ possible outcomes}}.$$



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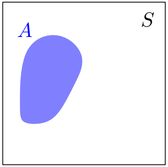

From this definition, $0 \leq P(A) \leq 1$.

Differents ways to state probabilities numerically:

- Day-to-day life: "This event is 70% likely."
- Scientific area: "This event has probability .7."

Probabilities: Example 1

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The sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

$$P(A) = \frac{\#A}{\#S} = \frac{3}{6} = \frac{1}{2}.$$

Probabilities: Example 2

You flip a fair coin twice. Compute the probability of getting exactly 1 heads.

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$$S = \left\{ \begin{array}{cccc} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4) \end{array} \right\}$$

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$$P(C) = \frac{\#C}{\#S} = \frac{10}{16} = \frac{5}{8}.$$

Further Examining Probabilities

- If an event cannot occur, then $P(A) = 0$.
  Example: $A = \{\text{getting heads and tails in one coin flip}\}$
- If an event must occur, then $P(A) = 1$.
  Example: $A = \{\text{getting heads or tails}\}$.

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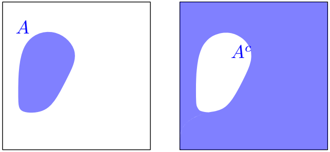
- Given an event A, its **complement** A^c is the set of all outcomes not in A but in the sample space ($A^c = S \setminus A$). In particular,

$$P(A^c) = 1 - P(A).$$



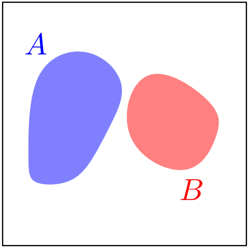
Example: If $A = \{\text{rolling an even number on a die}\}$, then

$$\begin{aligned} A^c &= \{\text{not rolling an even number}\} \\ &= \{\text{rolling an odd number on a die}\} \end{aligned}$$

The "or" Rule

If two events A and B are disjoint, then

$$P(A \text{ or } B) = P(A) + P(B).$$



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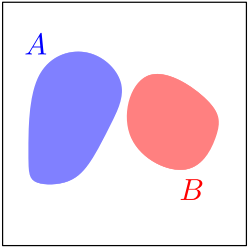

Generalization: If $A_1, A_2, \ldots, A_n$ are mutually disjoint events, then

$$P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$

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Let A be the even of five Heads:

$$A = \{HHHHH\}.$$

Let B be the event of exactly one Tail:

$$B = \{THHH, HTHHH, HHTHH, HHHTH, HHHHT\}.$$

S is the list of all words of length 5, formed with letters $\{H, T\}$. Hence,

$$\#S = 2 \times 2 \times 2 \times 2 \times 2 = 32.$$

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A and B <u>have no outcome in common</u>, therefore,

$$P(A \text{ or } B) = P(A) + P(B) = \frac{1}{32} + \frac{5}{32} = \frac{6}{32} = \frac{3}{16}.$$

Disjoint Events

Events A and B are **disjoint** if they share no common outcomes.

Examples of disjoint events:

- A = "Rolling an even number on a die"
  B = "Rolling a 5 on die"
- A = "Pulling a red or green M&M from a bag"
  B = "Pulling a blue or orange M&M from a bag"

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Examples of non-disjoint events:

- A = "Rolling an even number on a die"
  B = "Rolling a perfect square on a die"
- A = "Playing in the NFL"
  B = "Playing in the MLB"

If events are NOT disjoint, you should be able to think of an outcome that is common to both events. Try to do so with each of these.

It's All Gotta Add To 1

If you break up the sample space into disjoint events, the probabilities of theses events must add to 1 (=100%)

Example: Suppose the weather in SD is either Sunny, Cloudy, Rainy, or Snowy. If the first three have probabilities 0.85, 0.08 and 0.06, what is the probability of a snowy day?

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$$P(\text{Sunny}) + P(\text{Cloudy}) + P(\text{Rainy}) + P(\text{Snowy}) = 1,$$

So

$$0.85 + 0.08 + 0.06 + P(\text{Snowy}) = 1,$$

which yields $P(\text{Snowy}) = 0.01$.

Independence

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Two events A and B are said to be **independent** if knowing the outcome of one provides no useful information about the outcome of the other.

Examples of independent events:

- A = "Getting into UCSD"
  B = "Getting Tails on a coin flip"
- A = "Getting Heads on flip 1 of a coin"
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Examples of dependent events:

- A = "Getting into UCSD"
  B = "Getting into UCLA"
- A = "Getting a red card on top of a deck"
  B = "Getting a red card as the next card in a deck"

The “and” Rule

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Example: You toss a coin twice. What is the probability of getting two Heads?
A = "Getting heads on flip 1 of a coin"
B = "Getting heads on flip 2 of a coin"
Then,

$$\begin{aligned} P(\text{Two Heads}) &= P(\text{Heads on 1st toss and Heads on 2nd toss}) \\ &= P(\text{Heads on 1st toss}) \times P(\text{Heads on 2nd toss}) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}. \end{aligned}$$

Deciding If Events are Disjoint or Not

Disjoint or not? Ask yourself

Is there an outcome common to both events?

- **Yes:** Events are NOT disjoint
- **No:** Events are disjoint

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Example: $A = \{\text{Person uses Facebook}\}, B = \{\text{Person uses Twitter}\}$.

There are people on both services, so the events A and B are NOT disjoint.

Deciding If Events are Independent or Not

Independent or not?

1. Find $P(A)$
2. Find $P(A$ assuming you know event B has occured$)$

Do you get the same answer?

- **Yes:** Events are independent
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Example:

$P(\text{Facebook}) = 0.64$

$P(\text{Facebook given they use Twitter}) = 0.93$

These event are NOT independent (= dependent).

Practice

Let

$A = \{\text{Someone is a student at UCSD}\}$

$B = \{\text{Someone is part of Muir college}\}$

Events A and B are:

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2. Disjoint, dependent
3. Not disjoint, independent
4. Not disjoint, dependent

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Are there any outcomes in both events? Yes: a student at UCSD in Muir college. Not disjoint events.

P(student at UCSD) is small,
P(student at UCSD one you know he/she is in Muir college) = 1.
Different results means dependent events.

Practice

Let

$$A = \{\text{A student's first semester of undergrad is at UCSD}\}$$
$$B = \{\text{A student's first semester of undergrad is at UCLA}\}$$

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Your first semester can be at both schools: we have disjoint events.

$P(A) > 0$, $P(B) > 0$ and $P(A \text{ and } B) = 0$, so

$$P(A \text{ and } B) \neq P(A)P(B).$$

We have dependent events.

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Remark: Disjoint $\Rightarrow$ Dependent (The converse is not true!)

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The probability of a miss is $1 - 0.6 = 0.4$. As a consequence,

$$\begin{aligned} P(\text{3 misses in a row}) &= P(\text{Miss 1st and Miss 2nd and Miss 3rd}) \\ \text{using independence} \rightarrow &= P(\text{Miss 1st}) \times P(\text{Miss 2nd}) \times P(\text{Miss 3rd}) \\ &= 0.4 \times 0.4 \times 0.4 = 0.064. \end{aligned}$$

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You roll a green four-sided die (numbered 1-4) and note the outcome G. Do the same with a red four-sided die and note the outcome R. What is the probability that G/R is an integer?

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The values (G, R) that turn G/R integer are the 8 outcomes

$$1/1, 2/1, 2/2, 3/1, 3/3, 4/1, 4/2 \text{ and } 4/4.$$

The sample space has size $\#S = 4 \times 4 = 16$. Hence,

$$P(G/R \text{ integer}) = \frac{\#\{(G, R) \text{ such that } G/R \text{ integer}\}}{\#S} = 0.5.$$

Recap

- **"or" rule:** If two events A and B are disjoint,

$$P(A \text{ or } B) = P(A) + P(B).$$

Generalization: If $A_1, \ldots, A_n$ are disjoint,

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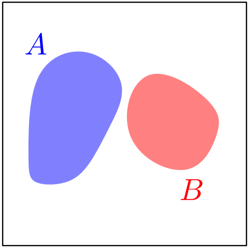

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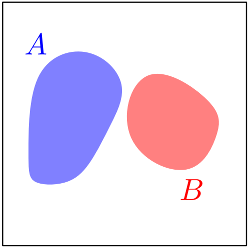

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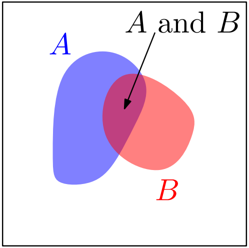
Generalization: If $A_1, \ldots, A_n$ are mutually independent,

$$P(A_1 \text{ and } \ldots \text{ and } A_n) = P(A_1) \times \ldots \times P(A_n).$$

Losing Disjointness

In general, for any two events A and B,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$



Losing Disjointness

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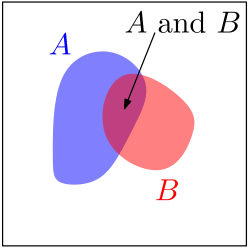


Caution: "or" is inclusive!

"A or B" = "1)A but not B, 2) B but not A, 3) A and B simultaneously".

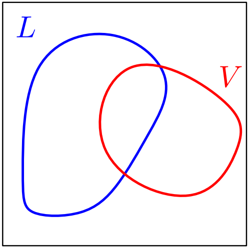
In everyday speech, "A or B" can mean this same thing (inclusive or), or sometimes it means just two ideas:
1) A but not B, 2) B but not A (exclusive or).

Non-Disjoint Events: Example

80% of college students like learning. 70% of college student like video games. 62% like both learning and video games.
What percent like learning or video games?

Let L be the event that a college student likes learning.
Let V be the event that a college student likes video games.

$$P(L \text{ or } V) = P(L) + P(V) - P(L \text{ and } V)$$



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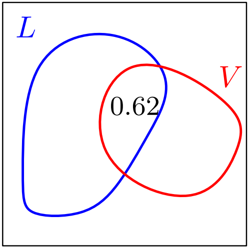

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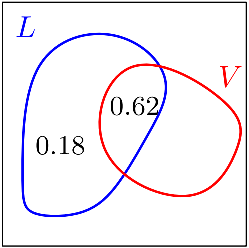

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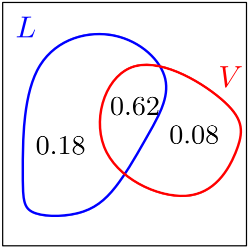

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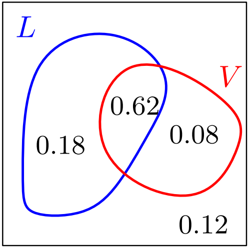

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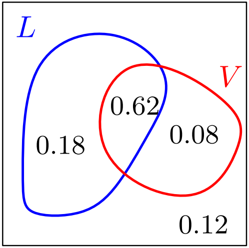
$$\begin{aligned} P(L \text{ or } V) &= P(L) + P(V) - P(L \text{ and } V) \\ &= 0.8 + 0.7 - 0.62 \\ &= 0.88. \end{aligned}$$



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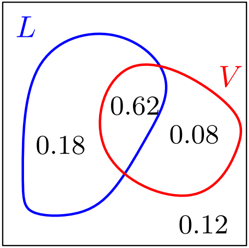
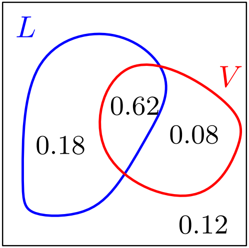


The picture on the right is called a **Venn diagram**.

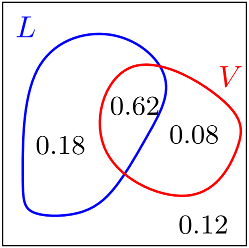
Non-Disjoint Events: Example



Describe in words the zone given by:

0.18

Non-Disjoint Events: Example



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0.18 People who like learning but not video games.

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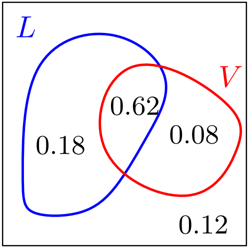

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0.08+0.62

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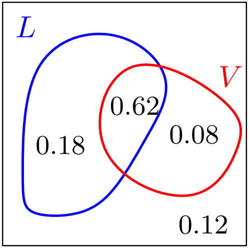

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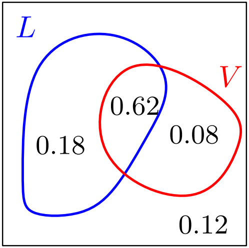

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0.12	

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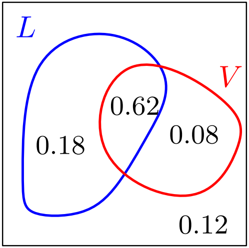

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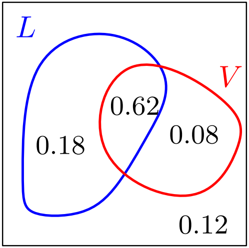

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0.18+0.08	

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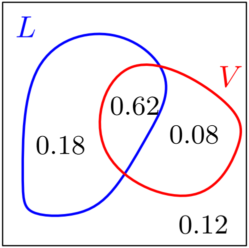
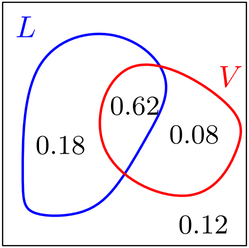


Describe in words the zone given by:

0.18	People who like learning but not video games.
0.08+0.62	People who like video games.
0.12	People who dislike learning and video games.
0.18+0.08	People who like learning only or video games only

Contingency Table



	Like video games	Dislike Video Games	Margin Totals
Like Learning	0.62	0.18	0.8
Dislike Learning	0.08	0.12	0.2
Margin Totals	0.7	0.3	1

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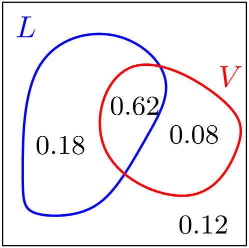

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Margin Totals	0.7	0.3	1

- **Joint Probabilities** are probabilities corresponding to two things happening simultaneously.
  Here: 0.62,0.18,0.08,0.12
- **Marginal Probabilities** are probabilities corresponding to the outcome of one variable.
  Here: 0.8,0.2 (for L) and 0.7, 0.3 (for V).

Contingency Table: Example

Among the students enrolled in a class,

- 41 are Junior with Biology major
- 87 are neither Junior nor with Biology major
- 131 have Major other than Biology
- 107 are not Junior

What is the probability that a randomly chosen student is Biology Major?

	Junior	Other Levels	Margin totals
Biology Major			
Other Major			
Margin Totals			

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Biology Major	41		
Other Major			
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Margin Totals		107	

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Other Major		87	131
Margin Totals		107	

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What is the probability that a randomly chosen student is Biology Major?

	Junior	Other Levels	Margin totals
Biology Major	41	20	61
Other Major		87	131
Margin Totals		107	192

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Biology Major	41	20	61
Other Major		87	131
Margin Totals		107	192

$$p = \frac{61}{192} \simeq 31.77\%$$