MATH-SHU 362

Recitation 1: Introduction of ODE

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Exercise 1. *Let us investigate the differential equation* $\frac{\mathrm{d}y}{\mathrm{d}t} + ty^2 = t^3$.

1. *Express the equation in the form* $\frac{\mathrm{d}y}{\mathrm{d}t} = F(y,t)$.
2. *Find the value* $F(y,t)$ *in the grid of* $[-5,5] \times [-5,5]$.
3. *Sketch the some solutions of the equation.*
4. *Describe the behaviors of this equation.*

Exercise 2. *Match the differential equations to the directional field. (See the complementary sheet.)*

Exercise 3. *Write down the classification of the following equations. Here* $y = y(t)$ *and* $f = f(t,x,y)$.

Equation	*Order*	*Linear/Nonlinear*	*ODE/PDE*
$y' + 2y = 0$			
$y' + 6y^2 = 0$			
$y^{(2000)} = t + t^2$			
$\frac{\mathrm{d}}{\mathrm{d}t}(y)^2 = y + t^2 + 7$			
$y^{(n)} - y^{(n-1)} = y^2$			
$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$			

Exercise 4. *Find the general solution of the given differential equations, and use it to determine how solutions behave as* $t \to \infty$.

1. $y' + 3y = t + e^{-2t}$,
2. $y' - 2y = te^{2t}$,
3. $y' - \frac{1}{t}y = 3\cos(2t), t > 0$,
4. $ty' - y = t^2 e^{-t}, t > 0$.

Exercise 5. *A pond initially contains 1,000,000 L of water and an unknown amount of an undesirable chemical. Water containing 0.01 grams of this chemical per liter flows into the pond at a rate of 300 L/h. The mixture flows out at the same rate, so the amount of water in the pond remains constant. Assume that the chemical is uniformly distributed throughout the pond.*

1. *Write a differential equation for the amount of chemical in the pond at any time.*
2. *How much of the chemical will be in the pond after a very long time? Does this limiting amount depend on the amount that was present initially?*