MATH-UA.0224@NYU

Midterm Exam

03/11/2020 Time: 75 min

Name: *Univ ID:*

In the following 5 questions, every question counts 5 points and please write down the main steps of the solution. The final score will be truncated by 20 i.e. $\min\{\text{score, } 20\}/20$.

Personal lecture notes are allowed for the exam.

Question 1 (5 pts)**.** *Develop Taylor expansion for* $f(x, y) = \sin(3x)e^{-y}+xy$ *at* $(0, 0)$ *to a precision* $o(|x|^2 + |y|^2)$.

Question 2 (5 pts)**.** *For* $F(x, y) = x^2 + 6xy + 4y^2$*, classify the its critical point* $(0, 0)$*. (Local minimum, local maximum or saddle point?)*

Question 3 (5 pts)**.** *Find the points of the ellipse* $x^2 + 2y^2 = 1$ *which are the closest or the farthest from the line* $x + y = 10$*.*

Question 4 (5 pts)**.** *Calculate the value of integrals* $\int_{-\infty}^{+\infty} e^{-x^2}\,dx$ *and* $\int_{-\infty}^{+\infty} xe^{-x^2}\,dx$*.*

Question 5 (5 pts)**.** *Let* $f(x) = \frac{x^2+1}{2}$*, and we construct an iteration* $x_{n+1} = f(x_n)$*. Then, for any* $x_0 \in [-1, 1]$*, prove that*

1. *This iteration admits a limit that* $\lim_{n\to\infty} x_n = x_*$*.*
2. *This limit* x_* *does not depend on the initial value.*
3. *Calculate* x_**.*