Talks

Workshop in Statistical Mechanics

February 2024

1 Alexander Glazman

Title: Random-cluster model on Z^2 at the transition point

Abstract: The random-cluster model is defined on subgraphs of Z^2 and has two parameters: cluster-weight $q > 0$ and edge-probability $0 < p < 1$. It is classical that, for each $q \geq 1$, the model undergoes a percolation phase transition when $p = p_c(q)$. Beffara and Duminil-Copin in 2010 computed $p_c(q)$, and later works established the type of the phase transition: it is continuous when $1 \leq q \leq 4$ and discontinuous when $q > 4$. The former is characterised by Russo-Seymour-Welsh estimates, while the latter asserts non-uniqueness of the infinite-volume Gibbs measure.

In this talk we revisit both parts of this diagram. When $1 \leq q \leq 4$, we give a new proof of continuity that does not use parafermionic observable, nor Bethe Ansatz. When $q > 4$, we establish invariance principle under Dobrushin boundary conditions - the interface converges to the Brownian bridge. Both arguments rely on the Baxter-Kelland-Wu correspondence that relates the random-cluster model to a certain height function (six-vertex model).

Remarkably, we obtain also some result when $q < 1$, though only at the self-dual point.

Joint works with Moritz Dober, Piet Lammers and Sebastien Ott.

2 Antoine Jego

Title: Thick points of 4d critical branching Brownian motion

Abstract: I will describe a recent work in which we prove that branching Brownian motion in dimension four is governed by a nontrivial multifractal geometry and compute the associated exponents. As a part of this, we establish very precise estimates on the probability that a ball is hit by an unusually large number of particles, sharpening earlier works by Angel, Hutchcroft, and Jarai (2020) and Asselah and Schapira (2022) and allowing us to compute the Hausdorff dimension of the set of "a-thick" points for each a ¿ 0. Surprisingly, we find that the exponent for the probability of a unit ball to be "a-thick" has a phase transition where it is differentiable but not twice differentiable at a = 2, while the dimension of the set of thick points is positive until a = 4. If time permits, I will also discuss a new strong coupling theorem for branching random walk that allows us to prove analogues of some of our results in the discrete case.

Joint work with Nathanael Berestycki and Tom Hutchcroft.

3 Istvan Prause

Title: Limit shapes and harmonic envelopes

Abstract: Limit shapes are deterministic surfaces in R^3 which arise in the macroscopic limit of discrete random surfaces associated to various probability models such as domino tilings, random Young tableaux or the 5-vertex model. The limit surface is a minimiser of a variational problem with a surface tension which encodes the local entropy of the model. I'll present a geometric "tangent plane method" which applies to general gradient variational problems, in particular also to non-free fermionic models. I'll illustrate the method in detail by considering random Young tableaux on an arbitrary diagram and also discuss related problems. The talk is based in part on joint work with Rick Kenyon.

4 Nicolas Curien

Title: Where do random trees grow leaves ?

Abstract : We study a model of random binary trees grown by the leaves in the style of Luczak and Winkler. If n is a uniform plane binary tree of size n, Luczak and Winkler, and later explicitly Caraceni and Stauffer, constructed a measure n such that the tree obtained by adding a cherry on a leaf sampled according to $\nu\tau n$ is still uniformly distributed on the set of all plane binary trees with size $n+1$. It turns out that the measure $\nu\tau n$, which we call the leaf-growth measure, is noticeably different from the uniform measure on the leaves of the tree n. In fact, we prove that, as $n \to \infty$ with high probability it is almost entirely supported by a subset of only $n0.8038...$ leaves.

5 Andrei Okunkov

Title: What is so special about special functions

Abstract:

Every generation of mathematical physicists invents a new all-encompassing and truly fundamental, in their opinion, way to interpret and generalize the special functions used by all previous generations of mathematical physicists. The current generation is, of course, no exception, and I will talk about some ways in which people like to think about objects like orthogonal polynomials, hypergeometric functions, etc.

6 Alexis Prevost

Title: Phase transition for the late point of random walk

Abstract: Consider a random walk on the d dimensional torus of size length N, $d > 2$, and let T be the average of the first time all the vertices of the torus have been visited by this walk. For $0 < \alpha < 1$, the α-late points are defined as the vertices not visited by this walk at time . It is known that for α large enough the α-late points are essentially i.i.d. on the torus, whereas this is not the case for α small enough. I will explain why this phase transition actually happens at some parameter $\alpha^\star > 1/2$, which can be explicitly written in terms of the Green function on the d-dimensional infinite lattice. I will also describe the law of these -late points in the non i.i.d. region $1/2 < \alpha < \alpha^\star$. Based on joint work with Pierre-François Rodriguez and Perla Sousi.

7 Emmanuel Kammerer

Title: Distances on the CLE(4), critical Liouville quantum gravity and 3/2-stable maps

Abstract: Random planar maps with high degrees are expected to have scaling limits related to the conformal loop ensemble (CLE) equipped with an independent Liouville quantum gravity (LQG). In the dilute case, where informally the degrees have finite expectations, Bertoin, Budd, Curien and Kortchemski established the scaling limit of the distances to the root. However, the scaling limit does not have an interpretation as a distance from the loops to the boundary in terms of LQG. I will focus on the critical case where the probability that a vertex has degree k is of order k^-2. In this case, the distances from the root to the high degree vertices satisfy a scaling limit, which is related to a quantum distance to the boundary on a CLE-decorated critical LQG introduced by Aru, Holden, Powell and Sun and to the conformally invariant exploration of the CLE from Werner and Wu.

8 Ulrik Hansen

Title: On the Number of Phase Transitions in the Ising Model.

Abstract: The Ising model is among the richest objects of study in statistical mechanics. One reason for this is its zoo of graphical representations, from the high temperature and random current expansions to its FK representation. As each of these models sees correlations in the Ising model through their connectivity properties, one very natural question suggests itself: Namely, whether their connectivity properties change jointly or not - for instance, whether the three models have an onset of percolation at the same temperature. In '18, Garet, Marchand and Marcovici showed that the high temperature expansion on Z^2 has a percolation phase transition at the Ising critical temperature, and by a result of Lupu and Werner, the same result follows for the random current. With the same techniques, however, one may prove that the high temperature expansion never percolates on the hexagonal lattice. Thus, the answer to the question outlined above is not as clear-cut as one might hope. In this talk, we will turn our attention to the higher dimensional case of Z^d for d at least 3. Here, we will use a coupling between the high temperature expansion and the random-cluster model due to Grimmett and Janson, which presents the former as a uniform even subgraph of the latter. This coupling then allows us to a) utilise a famous argument due to Kozma and Sidoravicius to show that the high temperature expansion exhibits long loops on the torus throughout the supercritical regime and b) show a mixing property of the uniform even graph, which allows us to compare the model on the torus to the model on Z^d. Based on joint work with Boris Kjær and Frederik Ravn Klausen.

9 Paul Dario

Title: Phase transition for the XY model on a percolation cluster.

Abstract: In this talk we will discuss the properties of a spin system known as the XY model. One of the important results of this model is the existence of a rather specific two-dimensional phase transition known as the Berezinsky-Kosterlitz-Thouless (BKT) transition. From a mathematical point of view, it was proved by Fröhlich and Spencer in 1981 and has recently been the subject of renewed activity following the works of Lammers, van Engelenburg and Lis, and Aizenman, Harel, Peled and Shapiro. We will present the model and some of its properties. We will then address the following question: Does the BKT transition persist when the XY model is subjected to random disorder? And in particular, is it observed for the model placed on an infinite cluster in supercritical Bernoulli percolation? Joint work with Christophe Garban.

10 Gady Kozma

Title: lace expansion on nilpotent groups.

Abstract: We survey the lace expansion method and what is needed to make it work on groups. We will explain the statement of the central limit theorem for nilpotent, torsion free groups and how it plays a role in the lace expansion scheme. Joint work with Erwin Bolthausen and Remco van der Hofstad.

11 Rémy Mahfouf

Title: From the planar Ising model to the Lorentz Geometry

Abstract : While looking for a natural procedure to embed graphs equipped with the Ising model into the plane, Chelkak (see also counterparts on the dimers side in his works with Russkikh and Laslier) discovered the unexpected appearance of the Lorentz geometry in the scaling limit of the Ising model. In particular, one can construct explicitly planar Ising models, which are critical or near critical, whose scaling limit is now expressed in terms of *massive fermions*

on some space-like surfaces of the Minkowski space $R^{2,1}$. The mean curvature of the associated surface plays the mass and appears to play the role of the correlation length of the model. His meta framework provides a general setup where one can expect and prove convergence statement, namely the convergence (in smooth domains) of FK-observables and the convergence of the energy correlation, similarly to the original works of Smirnov, and Hongler-Smirnov. Based on the PhD thesis of the author, a recent paper and on ongoing works with Sung-Chul Park.

12 Romain Panis

Title: New lower bound for the (near) critical Ising model's two-point function

Abstract: We study the nearest-neighbour Ising model on Z^d with $d \geq 3$ and obtain a new lower bound on its two-point functions at (and near) criticality. Together with the classical infrared bound, this bound turns to up-to constant estimates when $d \geq 5$. When $d = 4$, we prove an "almost" sharp lower bound corrected by a logarithmic factor. As a consequence of these results, we obtain that $\eta = 0$ when $d \geq 4$, where η is the critical exponent associated with the decay of the model's two-point function at criticality. When $d = 3$, we improve previous results and show that $\eta 1/2$.

As a byproduct of our methods, we derive the blow-up at criticality of the so-called bubble diagram when d=3,4. The proof is based on the random current representation.

If time permits, we will discuss some applications of this result to the study of the Ising model on the torus in dimension $d \geq 5$.

13 Baptiste Cercle

Title: Toda Conformal Field Theories and higher-spin symmetry

Abstract: Conformal invariance is a feature that (most of the time conjecturally) arises for a large class of models of statistical physics at criticality. To address the issue of understanding the conformal field theory (CFT) thus defined, Belavin-Polyakov-Zamolodchikov designed in 1984 a general method for solving such a theory, dubbed conformal bootstrap.

However there is a large class of models, such as the tricritical three-states Potts model, that enjoy in addition to conformal invariance an enhanced level of symmetry called higher-spin symmetry. Capturing this additional feature led to the introduction by Zamolodchikov of W-algebras, which are Vertex Operator Algebras containing the Virasoro algebra.

In this talk we will explain how this higher-spin symmetry manifests itself for Toda CFTs, generalizations of Liouville CFT that enjoy this higher level of symmetry. To do so we will rely on a probabilistic definition of these theories. To be more specific, we will first construct a representation of W-algebra as operators acting on the Hilbert space of the theory. We will then consider the purely probabilistic problem of computing two-point functions of the theory and its connections to asymptotics of Whittaker functions. Finally we will present some results and works in progress related to the computation of generic correlation functions, and shed light on the AGT correspondence for the W-conformal blocks of the theory.

14 Alexander Bobenko

We start by showing a 63 minutes film (made professionally for mathematicians) about the resolution of the Bonnet Problem.

Title: How discrete integrable systems helped to solve two classical problems in differential geometry

Abstract: We consider a classical problem in differential geometry, known as the Bonnet problem, whether a surface is characterized by a metric and mean curvature function. We explicitly construct a pair of immersed tori that are related by a mean curvature preserving isometry. This resolves a longstanding open problem on whether the metric and mean curvature function determine a unique compact surface. Moreover, since these tori are real analytic, this resolves a second longstanding open problem on whether real analyticity of the metric already determines a unique compact immersion. Discrete integrable systems and discrete differential geometry are used to find crucial geometric properties of surfaces. This is a joint work with Tim Hoffmann and Andrew Sageman-Furnas.

15 Stanislav Smirnov

Title: Coulomb gas and lattice models

Abstract: Even before the introduction of Conformal Field Theory by Belavin, Polyakov and Zamolodchikov, many exponents and dimensions of 2D lattice models at criticality were derived by den Nijs and Nienhuis using Coulomb gas techniques. The latter postulate (unrigorously) that height functions of lattice models converge to the Gaussian Free Field, which is in many ways mysterious, in particular was never formulated in the presence of a boundary. We will discuss possible formulations and their relations to CFT, SLE and conformal invariance of critical lattice models.

16 Christophe Garban

Title : Harmonic analysis of Gaussian multiplicative chaos

Abstract : In a joint work with Vincent Vargas, we initiate the harmonic analysis of Gaussian multiplicative chaos (GMC) on the circle, i.e. the study of its Fourier coefficients. In particular, we show that almost surely GMC is a so-called Rajchman measure which means that its Fourier coefficients converge to 0 when the frequency goes to infinity. We supplement this result with a

convergence in law for the rescaled Fourier coefficient towards an explicit law. I will review all the necessary background on GMC in order to apprehend the asymptotic behaviour of its Fourier coefficients.