ERRATUM TO *INTERMEDIATE JACOBIANS AND RATIONALITY OVER ARBITRARY FIELDS*

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The following statement is [BW23, Theorem 4.13]. We have become aware that its proof is invalid when the ground field has characteristic 2. This theorem is not used anywhere in [BW23] but it is of general interest. We correct its proof below.

Theorem. *Let k be a field. Fix $N \geq 3$, and let $X \subset \mathbf{P}^N_k$ be a smooth cubic hypersurface. The following assertions are equivalent:*

(i) The variety X is separably k-unirational.
(ii) The variety X is k-unirational.
(iii) One has $X(k) \neq \varnothing$.

The proof of the implication (ii)⇒(i) given in [BW23] proceeds, after a reduction to the case of a cubic surface over an infinite field, by constructing a rational map ϕ from a rational surface to X and by verifying that ϕ is separable. The map ϕ is erroneously claimed to have degree 9 (it has degree 6) and therefore to be separable if k has characteristic 2; a proof of separability is then given in [BW23] under the assumption that the characteristic of k is not 2. To fill this gap, we provide an argument for the separability of ϕ that does not exclude characteristic 2.

Proposition. *Let k be an algebraically closed field. Let $X \subset \mathbf{P}^3_k$ be a smooth cubic surface. For general $(x, y) \in (X \times X)(k)$, the third intersection point map $\phi : C_x \times C_y \dashrightarrow X$, where C_x (resp. C_y) denotes the hyperplane section of X that is singular at x (resp. y), is dominant and separable.*

We recall that C_x (resp. C_y) is an integral rational curve with a double point at x (resp. y) (see [Kol02, Proposition 3.1]).

Proof. Let $(x', y') \in (C_x \times C_y)(k)$ be general. Assume for contradiction that ϕ is not étale at (x', y'). This exactly means that the lines $T_{y'}C_y$ and $T_{x'}C_x$ meet at some point z. As (x, y) is general, one has $y \notin T_xX$, so the planes T_xX and T_yX intersect along a line D not containing y. In addition, as x' is general, one has $x' \notin D$. It follows that $z = D \cap T_{x'}C_x$ is independent of y'. The point z is therefore characterized by the property that it lies on all the tangents to C_y at smooth points. By symmetry, it also lies on all the tangents to C_x at smooth points, and hence it is also independent of the choice of x and of y.

We have now shown that the point z belongs to the tangent hyperplane to X at a general point. The projective dual $\check{X} \subset \check{\mathbf{P}}^3_k$ of X is therefore contained in the projective dual of z, which is a hyperplane in $\check{\mathbf{P}}^3_k$.

It follows that X is not ordinary in the sense of [Kle86, I-6], and we deduce from [Kle86, Theorem 17 (iii)⇒(i)] that no hyperplane section of X admits a nondegenerate double point. In particular, no hyperplane section of X consists

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of a line and a conic intersecting transversally. It therefore follows from [Hom97, Theorem 1.1 (iii)⇒(i)] that k has characteristic 2 and that X is projectively equivalent to the Fermat cubic surface $F \subset \mathbf{P}^3_k$, defined by $a^3 + b^3 + c^3 + d^3 = 0$. As the tangent plane to F at $[a : b : c : d] \in F(k)$ has coordinates $[a^2 : b^2 : c^2 : d^2]$ in $\check{\mathbf{P}}^3_k$, the projective dual $\check{F}$ of F is the Fermat cubic surface in $\check{\mathbf{P}}^3_k$. In particular, it is not contained in a hyperplane of $\check{\mathbf{P}}^3_k$, and hence neither is $\check{X}$. This is the required contradiction. □

References

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