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# Midterm

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To do a later question in a problem, you can always assume a previous question even if you have not answered it.

**Problem 1 :**

1. Show that a group homomrphism $f$ is injective if and only if $\ker(f) = \{1\}$.
2. Define what a $k$-cycle in $S_n$ is.
3. Show that two disjoint cycles commute.

**Problem 2 :**
Let $G$ be a group whose only subgroups are $\{1\}$ and $G$. Show that $G$ is isomorphic to $\{1\}$ or $\mathbb{Z}/p\mathbb{Z}$ for some prime $p$.

**Problem 3 :**

1. Let $A = \{1, s, r^2, sr^2\} \subset D_8$, compute $C_{D_8}(A)$ and $N_{D_8}(A)$.
2. Show that $Z(D_{2n}) = \{1\}$ if $n$ is odd.

**Problem 4 :**
Let $G$ be a group. For all $g \in G$, we define $f_g : G \to G$ by $f_g(x) := g \cdot x \cdot g^{-1}$.

1. Show that $f_g$ is a group automorphism.
2. Show that $\theta : g \mapsto f_g$ is a group homomorphism from $G$ into $\mathrm{Aut}(G)$.
3. Show that $\ker(\theta) = Z(G)$