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Midterm

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To do a later question in a problem, you can always assume a previous question even if you have not answered it.

Problem 1 :
These questions were covered in class.

1. Let G be a group acting on a set X and let $x \in X$. Define $\mathrm{Stab}_G(x)$ and show that it is a subgroup of G.
2. Show that the kernel of a group homomorphism is normal.
3. State the first isomorphism theorem.

Problem 2 :
Let G be a group. We define $X := \{(x_0, x_1 \ldots, x_{p-1}) : \prod_{i=0}^{p-1} x_i = 1\}$.

1. Show that $|X| = |G|^{p-1}$.
2. Show that if $(x_0, \ldots, x_{p-1}) \in X$, then for all $0 < n < p$, we have:
$$(x_n, x_{n+1} \ldots, x_{p-1}, x_0, \ldots, x_{n-1}) \in X.$$
3. Let $\sigma \in S_p$ be the cycle $(0\,1 \ldots p-1)$. Show that
$$n \star (x_0, \ldots, x_{p-1}) := (x_{\sigma^n(0)}, \ldots, x_{\sigma^n(p-1)})$$
defines an action of $\mathbb{Z}$ on X.
4. Show that for all $x \in X$, $p\mathbb{Z} \subseteq \mathrm{Stab}_{\mathbb{Z}}(x)$.
5. Show that for all $x \in X$, the orbit of x has size 1 or p.
6. Show that the orbit of x has size 1 if and only if $x = (x_0, x_0, \ldots, x_0)$.
7. Assume that p divides $|G|$. Show that p divides the number of orbits of size 1. Deduce (without using Cauchy's theorem) that there is an element of order p in G.