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Midterm (Lecture 003)

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To do a later question in a problem, you can always assume a previous question even if you have not answered it.

Problem 1 (Translation action) **:**
Let G be a finite group of order m and $g \in G$ be an element of order n. Let $G = \{g_0, \dots, g_{m-1}\}$ and let $\sigma_g : \{0, \dots, m-1\} \to \{0, \dots, m-1\}$ be the map such that $\sigma_g(i) = j$ where $g \cdot g_i = g_j$. Recall that $\varepsilon : S_m \to \{1, -1\}$ is the sign of a permutation.

1. Show that σ_g is a bijection.
2. Show that σ_g is a disjoint product of n-cycles.
3. Show that $\varepsilon(\sigma_g) = (-1)^{(n-1)m/n}$.

Problem 2 (Groups of order 15) **:**
Let G be a group of order 15.

1. Show that there exists a and $b \in G$ such that a is order 3, b has order 5 and $G = \langle a, b\rangle$.
2. Show that $aba^{-1} \in \langle b\rangle$.
3. Show that $aba^{-1} = b$.
4. Show that $G \cong \mathbb{Z}/15\mathbb{Z}$.