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Homework 2

Due September 11th

The questions indicated as Harder will not be taken in account when grading.

Problem 1 (Order) **:**

1. Find the order of every element in $(\mathbb{Z}/18\mathbb{Z}, +)$ and of every element of $((\mathbb{Z}/18\mathbb{Z})^\star, \cdot)$. (You should start by giving a list of the elements of $\mathbb{Z}/18\mathbb{Z}$ that have a multiplicative inverse; there are six of them).
2. Let G be a group, a, $b \in G$. Show that the order of $a \cdot b$ is equal to the order of $b \cdot a$.
3. Let G be a group such that every (non identity) element has order 2. Show that G is abelian.

Problem 2 (Permutations) **:**

1. Let $\gamma \in S_n$ be an k-cycle. What are the $i \in \mathbb{Z}$ such that γ^i is a k-cycle.
2. Show that every element of S_n can be written as an arbitrary product of the elements (01) and $(01\dots n-1)$ (we say that (01) and $(01\dots n-1)$ generate S_n).
3. (Harder) Let $\tau = (0i)$ for $0 \leqslant i < n$ and $\gamma = (01\dots n-1)$. Find a necessary and sufficient condition on i so that τ and γ generate S_n.
4. Show that if Ω is an infinite set then S_Ω is infinite.
5. (Harder) Assume that Ω is countable, show that S_Ω has cardinality continuum (i.e. is in bijection with 2^Ω).

Problem 3 :
Let G be a group whose cardinal is even.

1. Let $X = \{g \in G : g \neq g^{-1}\}$. Show that $|X|$ is even.
2. Show that there is a element of order 2 in G.

Problem 4 :
Let $(G, \cdot)$ and $(H, \star)$ be to groups. We define $(g_1, h_1) \circ (g_2, h_2) \coloneqq (g_1 \cdot g_2, h_1 \star h_2)$.

1. Show that $(G \times H, \circ)$ is a group.
2. Show that $G \times H$ is Abelian if and only if G and H are.
3. (Harder) Let $(G_i)_{i \in I}$ be a collection of groups. Show that $\prod_{i \in I} G_i$ with the coordinatewise operation, i.e. $(g_i)_{i\in I} \cdot (h_i)_{i\in I} = (g_i \cdot h_i)_{i\in I}$, is a group.