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Homework 6

Due October 16th

Problem 1 :
Let R be a commutative ring such that $1 \neq 0$ and $S \subseteq R$ be closed under multiplication (i.e. $\forall x, y \in S$, $xy \in S$) and contain 1. We define the relation E on $R \times S$ by $(a,s)E(b,t)$ if and only if there exists $x \in S$ such that $xat = xbs$.

1. Show that E is an equivalence relation.
2. Let R_S denote the set $(R \times S)/E$ (it is the set of E-classes). If $(a,s) \in R \times S$, we denote by $\overline{(a,s)} \in R_S$ the E-class of (a,s). Show that the map $(\overline{(a,s)}, \overline{(b,t)}) \mapsto \overline{(ab, st)}$ is well defined. We denote this map $\star$.
3. Show that the map $(\overline{(a,s)}, \overline{(b,t)}) \mapsto \overline{(at+bs, st)}$ is well defined. We denote this map $\square$.
4. Show that $(R_S, \square, \star)$ is a commutative ring.
5. Show that the map $a \mapsto \overline{(a,1)}$ is a ring homomorphism from $\varphi : R \to R_S$.
6. Show that if S contains 0 then R_S is the trivial ring.
7. Show that φ is not injective if and only if S contains a zero-divisor.
8. Show that $R \smallsetminus \{0\}$ is closed under multipication if and only if R is an integral domain.
9. Assume that R is an integral domain. Show that $R_{(R \smallsetminus \{0\})}$ is a field.
10. Show that $\mathbb{Z}_{(\mathbb{Z} \smallsetminus \{0\})}$ is isomorphic (as a ring) to $\mathbb{Q}$.