Ranked groups - The return / Groupes rangés : le retour

All talks will take place in Salle Fokko du Cloux (Institut Camille Jordan, Université Lyon 1), or on visio. They will also be transmitte online on Zoom.

Timetable

Thursday September 23, 2021

10am	**Opening**: **Simon Masnou** (head of the Institut Camille Jordan), and **Michel Broué**
10h45	**Bruno Poizat** (Lyon 1), *La Conjecture d'Algebricité, dans une perspective historique / The Algebraicity Conjecture, in a historical perspective*
12h	**Ayse Berkman** (Mimar Sinan, Turkey), *Generically Multiply Transitive Actions on Solvable Groups in the Finite Morley Rank Context*
13h	Lunch break
14h30	**Olivier Frécon** (Poitiers), *Groupes harmonieux (partie 1 : une analyse de Lascar des groupes de rang de Morley fini revisitée)*
15h45	**Rachad Bentbib** (Poitiers) *Harmonious Groups (part 2: Their structure inside groups of finite Morley rank)*
17h15	**Rahim Moosa** (Waterloo, Canada), *From Borovik-Cherlin to bounding non-minimality*

Friday September 24, 2021

9h	**Katrin Tent** (Münster, Germany), *Mock hyperbolic reflection spaces and Frobenius groups of finite Morley rank*
10h15	**Samuel Zamour** (Lyon 1), *Dimensional Quasi-Frobenius Groups*
12h	**Gregory Cherlin** (Rutgers, USA), *Finite primitive binary permutation groups*
13h	Lunch break
14h30	**Alexandre Borovik** (Manchester, UK), *Actions of finite groups and simple algebraic groups on abelian groups of finite Morley rank*
15h45	**Ulla Karhumäki** (Helsinki, Finland), *Small groups of finite Morley rank with a tight automorphism*
17h15	**Joshua Wiscons** (California State U. Sacramento, USA), *Minimal representations of Sym(n) and Alt(n)*

ABSTRACTS

Rachad Bentbib (Poitiers) *Harmonious Groups (part 2: Their structure inside groups of finite Morley rank)*

This work is joint with Olivier Frécon.
Our main motivation is the caracterisation of $\aleph_1$-categorical groups among the pure groups.
We investigate how the harmonious interpretable subgroups with respect to non-analogous strongly minimal sets interact with each other inside a group G of finite Morley rank.
We show that, if G is connected, the group $G/Z(G)$ is a finite direct product of harmonious interpretable groups and the derived subgroup of G is a central product of harmonious interpretable groups, noting that a harmonious group in a language of cardinality κ is λ-categorical for every $\lambda > \kappa$.
Furthermore, we hypothesize that any group G of finite Morley rank is a central product of nonnecessarily interpretable harmonious groups, and we present several results in this direction.

Ayşe Berkman (Mimar Sinan, Turkey), *Generically Multiply Transitive Actions on Solvable Groups in the Finite Morley Rank Context*

I shall talk about a joint work with Alexandre Borovik on the following problem posed by Borovik and Cherlin in 2008.

Problem. Let G be a connected group of finite Morley rank acting faithfully, definably, and generically m-transitively on a connected abelian group V of finite Morley rank, where $m \geq rk(V)$. Show that $m = rk(V)$ and the action $G \curvearrowright V$ is equivalent to the natural action $\mathrm{GL}_m(F) \curvearrowright F^m$ for some algebraically closed field F.

Alexandre Borovik (Manchester, UK), *Actions of finite groups and simple algebraic groups on abelian groups of finite Morley rank*

I will explain some ideas of a proof of the following result

Theorem. Let $G = G_1 \times \cdots \times G_m$ where each $G_i = G_i(K_i)$ is the group of points over some algebraically closed field K_i of characteristic $p > 0$ of a simple algebraic group defined over K_i. Assume that G acts faithfully, definably and irreducibly on a connected elementary abelian p-group V of finite Morley rank.
Then all K_i are definably isomorphic to the same field K and V has a structure of a finite dimensional K-vector space compatible with the action of G, and G is a Zariski closed subgroup of $\mathrm{GL}_K(V)$.

Gregory Cherlin (Rutgers, USA), *Finite primitive binary permutation groups*

Recently Gill, Liebeck, and Spiga have completed the classification of finite primitive binary permutation groups, confirming a long-standing conjecture. I'll discuss this work, and some related problems. We might want to consider, as well, what transpires in the category of ranked groups.

Olivier Frécon (Poitiers), *Groupes harmonieux (partie 1 : une analyse de Lascar des groupes de rang de Morley fini revisitée)*

/This work is joint with Rachad Bentbib./
In a structure of finite Morley rank, we say that two interpretable strongly minimal sets X and Y are analogous if there is another strongly minimal set U and two interpretable maps $f : U \to X$ and $g : U \to Y$ with cofinite images. A structure of finite Morley rank is said to be harmonious if all its interpretable strongly minimal sets are analogous. In the context of groups, this notion is preserved by elementary equivalence and it is expected that a group of finite Morley rank is harmonious if and only if it is $\aleph_1$-categorical.
We will show that any strongly minimal structure is harmonious and that any group of finite Morley rank has finitely many equivalence classes of strongly minimal sets.

Ulla Karhumäki (Helsinki, Finland), *Small groups of finite Morley rank with a tight automorphism*

The famous Cherlin-Zilber conjecture proposes that any infinite simple group of finite Morley rank is isomorphic to a Chevalley group over an algebraically closed field.
In my talk, I will first introduce a recent approach towards this conjecture, which was suggested by P. Ugurlu and is based on the notion of a tight automorphism. I will then discuss a result joint with Ugurlu, stating that any "small" infinite simple group of finite Morley rank with a tight automorphism whose fixed-point subgroup is pseudofinite is isomorphic to the Chevalley group PSL_2 over an algebraically closed field of characteristic different from 2.

Rahim Moosa (Waterloo, Canada), *From Borovik-Cherlin to bounding nonminimality*

In this talk on joint work with James Freitag, I will explain how the truth of the Borovik-Cherlin conjecture, applied to binding group actions in certain totally transcendental theories of interest, leads to a useful bound on a degree that measures how far a finite-rank type is from being of rank 1. This is motivated by the search for new methods to verify the strong minimality of an algebraic differential equation.

Bruno Poizat (Lyon 1), *La Conjecture d'Algebricité, dans une perspective historique*

Je décrirai l'apparition de la Conjecture dans les années 70, en me réferant aux documents originaux, et je poserai quelques questions qui, à ma connaissance, sont ouvertes et qui me semblent plus abordables que la Conjecture elle-même. Cet exposé n'est pas une introduction au sujet : je supposerai que mon auditoire est assez familier des groupes de rang de Morley fini.

The Algebraicity Conjecture, in a historical perspective

I will describe the beginnings of the Conjecture in the '70, with reference to the original documents, and will ask some questions which, to my best knowledge, are open, but seem to me simpler than the Conjecture itself. This talk is not an introduction to the subject: I will assume from the audience a certain familiarity with the groups of finite Morley rank.

Katrin Tent (Münster, Germany), *Mock hyperbolic reflection spaces and Frobenius groups of finite Morley rank*

A Frobenius group is a group G together with a proper nontrivial malnormal subgroup H. A classical result due to Frobenius states that finite Frobenius groups split, i.e. they can be written as a semidirect product of a normal subgroup and the subgroup H. It is an open question if this holds true for groups of finite Morley rank, and the existence of a non-split Frobenius group of finite Morley rank would contradict the Algebraicity Conjecture. We use mock hyperbolic reflection spaces, a generalization of real hyperbolic spaces, to study Frobenius groups of finite Morley rank. We show that the involutions in a connected Frobenius group of finite Morley rank and odd type form a mock hyperbolic reflection space. These spaces satisfy certain rank inequalities and we conclude that connected Frobenius groups of odd type and Morley rank at most 6 split. Moreover, by using a construction from the theory of K-loops we show that if G is a connected Frobenius group of degenerate type with abelian complement, then G can be expanded to a group whose involutiuons almost form a mock hyperbolic reflection space.

Joshua Wiscons (California State U. Sacramento, USA), *Minimal representations of Sym(n) and Alt(n)*

We discuss the problem of determining the minimal faithful representations of $\mathrm{Sym}(n)$ and $\mathrm{Alt}(n)$ (on not necessarily abelian groups) in a setting of finite Morley rank, also reviewing connections to permutation groups possessing a high degree of generic transitivity. Two specific, rather opposite instances of the problem will be highlighted: actions on abelian groups and actions on nonsolvable groups without involutions. In the former case, we work in a new, natural context that covers at once the classical, finitary case as well as modules definable in an o-minimal or finite Morley rank setting. This represents joint work with Tuna Altınel, Barry Chin, Luis Jaime Corredor, Adrien Deloro, and Andy Yu.

Samuel Zamour (Lyon 1), *Dimensional Quasi-Frobenius Groups*

Following the work of A. Deloro and J. Wiscons, we study quasi-Frobenius groups (QF), namely pairs of connected groups $C < G$ such that $N_G(C)/C$ is finite and C intersects any distinct conjugate trivially. Principal examples are connected Frobenius groups, but also the classical groups $\mathrm{GA}_1(C)$, $\mathrm{PGL}_2(C)$ (ranked universe) and $\mathrm{SO}_3(R)$ (o-minimal universe). We consider essentially ranked (QF) with involutions but we will say some words about the o-mimimal case. Focusing on translations, i.e., products of involutions, we study the structure of Borel subgroups and we derive some classification results, notably an identification theorem for $\mathrm{PGL}_2(K)$, with K an algebraically closed field of characteristic different from two.